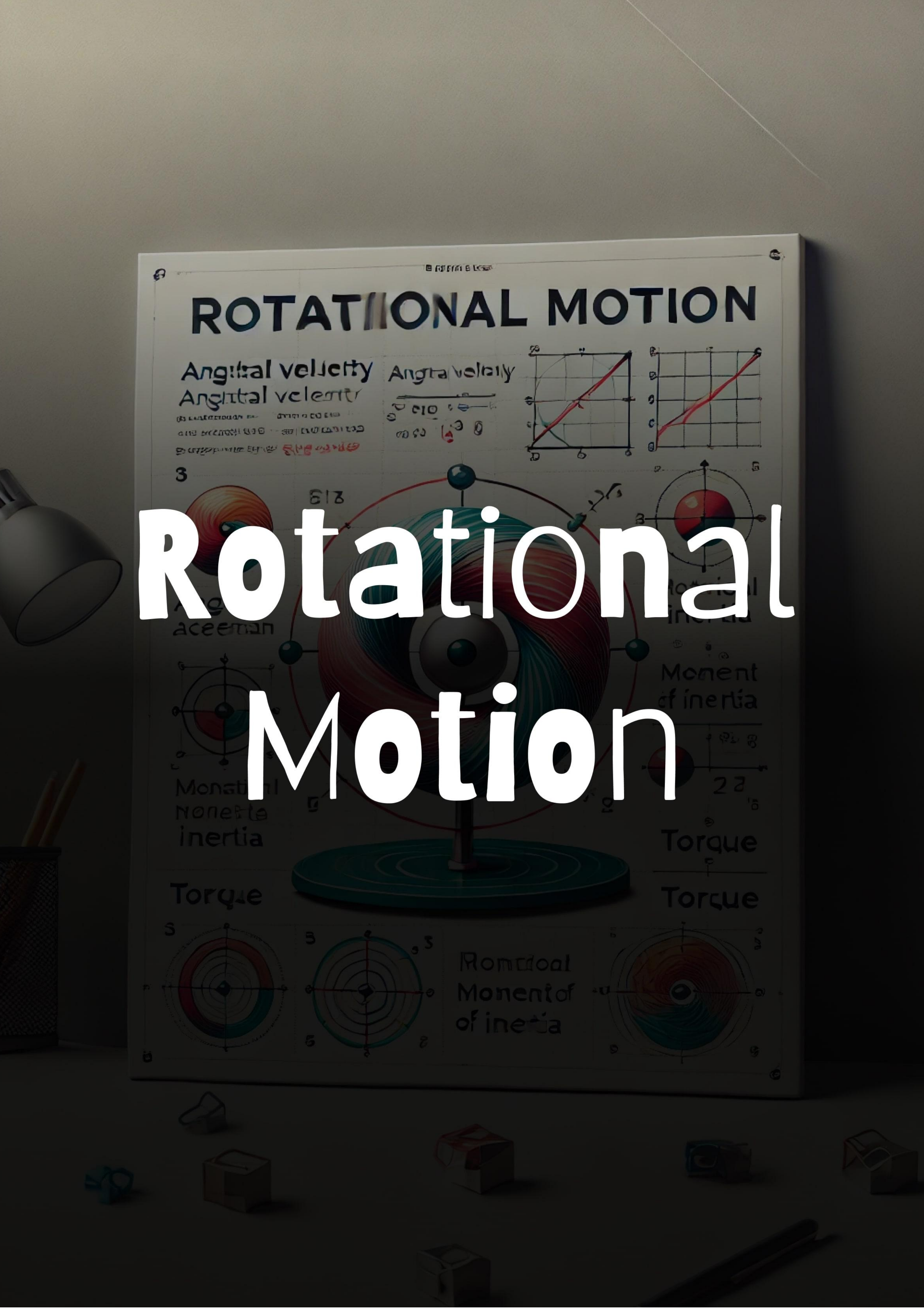


Rotational Motion

Rotational Motion

→ Property of the body by virtue of which it oppose cause of change in rotational state of motion

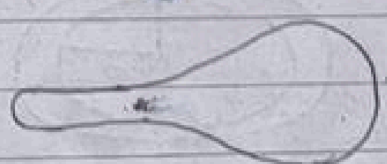
unit: kg-m²

Dimension: $M^1L^2T^0$ Scalar → oppose Torque

Depend upon
⊙ mass and Distribution of mass and
⊙ axis of rotational.

Doesn't Depend upon
Torque
angular acceleration
work

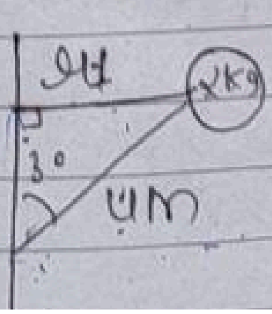
★ Mass jitni door Hoga MOI jyada Hoga

(1)  I_1

$I_1 > I_2$

 I_2

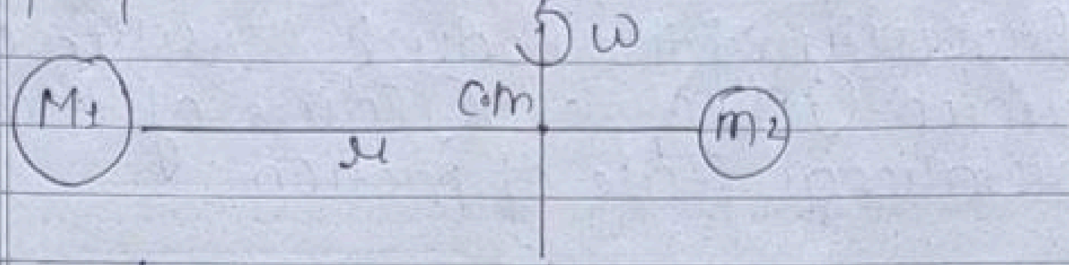
$[I = m(r_1)^2]$

(2) 

$\sin 30° = \frac{P}{H} = \frac{r_1}{4}$

$\frac{1}{2} = \frac{r_1}{4}$ $r_1 = 2$

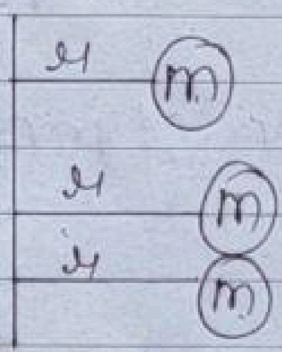
Q) Find MOI about Centre of mass and perpendicular to line.



$$I = \left(\frac{m_1 m_2}{m_1 + m_2}\right) r^2$$

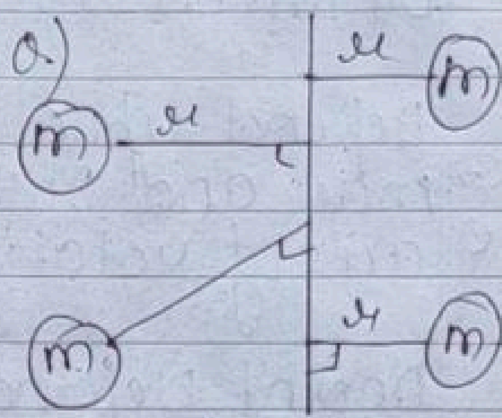
If $m_1 = 0$, $I = 0$ if $m_2 = 0$, $I = 0$

Q)



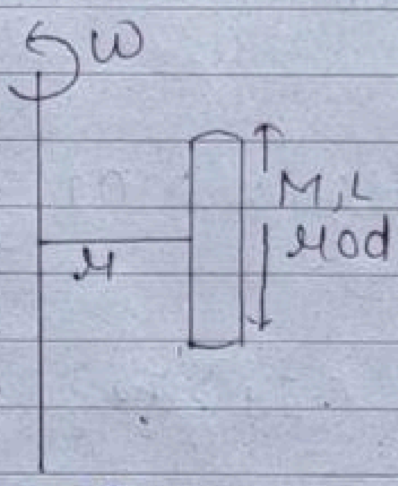
$I = 4mr^2$

Q)



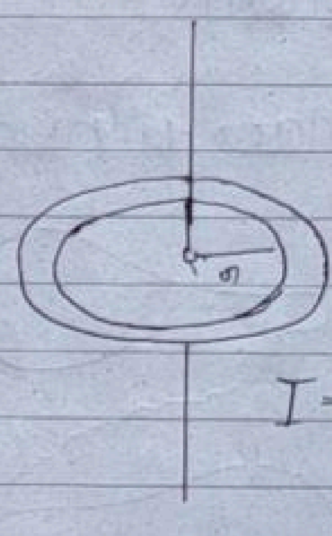
$I = 4mr^2$

Q)



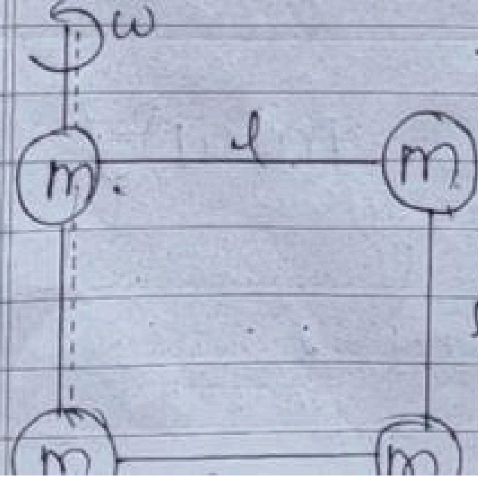
$I = Mr^2$

Q)



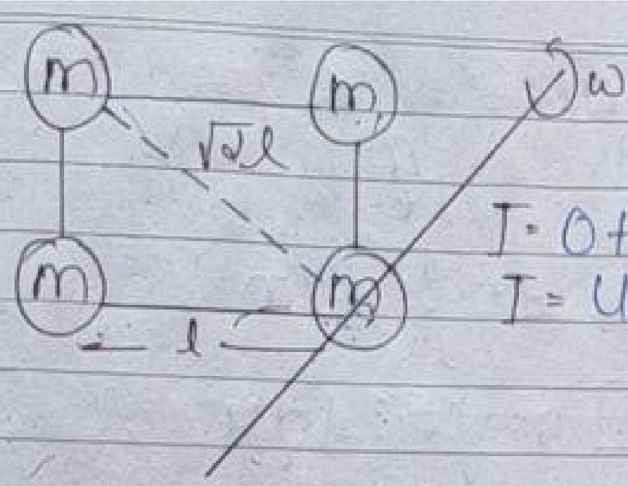
$I = MR^2$

Q)



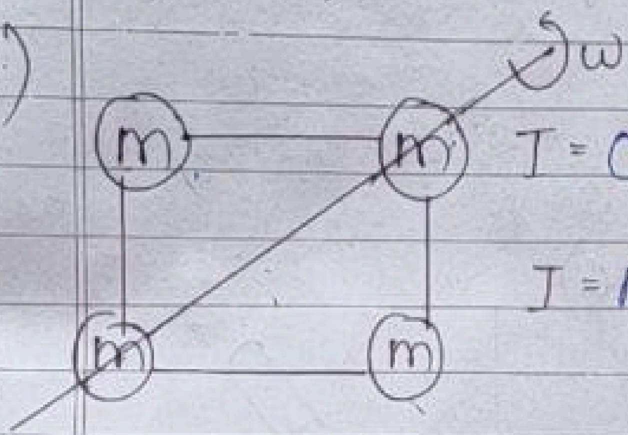
$$I = 0 + 0 + ml^2 + ml^2$$
$$= 2ml^2$$

Q.)



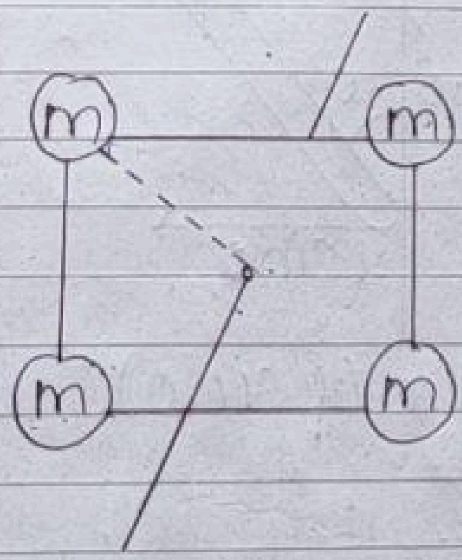
$$I = 0 + ml^2 + ml^2 + m(\sqrt{2}l)^2$$
$$I = 4ml^2$$

Q.)



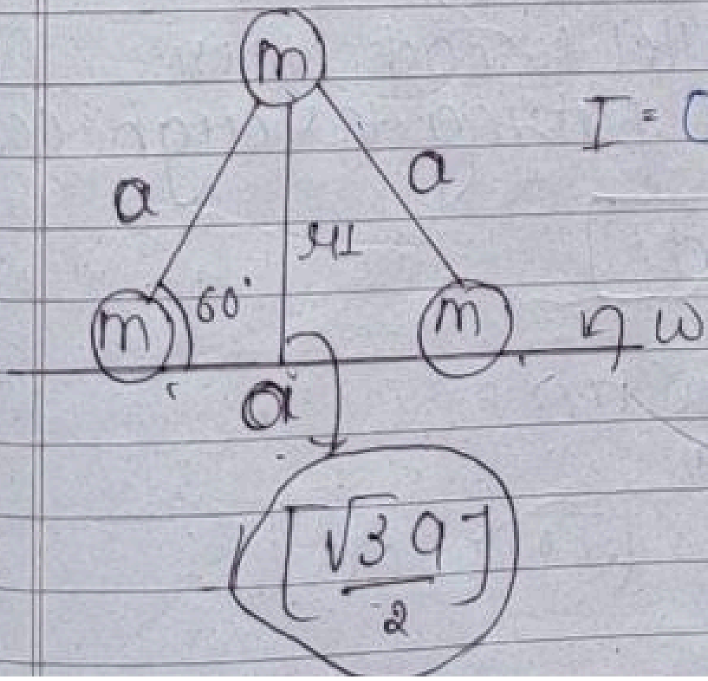
$$I = 0 + 0 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2$$
$$I = ml^2$$

Q.)



$$I = 4m\left(\frac{l}{\sqrt{2}}\right)^2 = 2ml^2$$

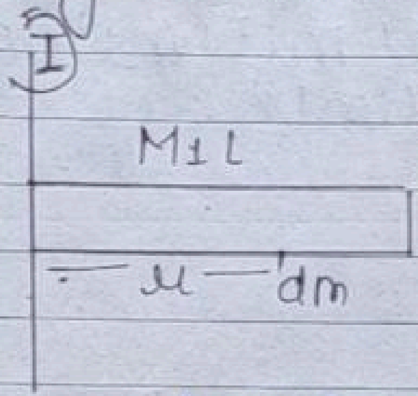
Q.)



$$I = 0 + 0 + m\left(\frac{\sqrt{3}a}{2}\right)^2$$
$$= \frac{m3a^2}{4}$$

Continuous mass

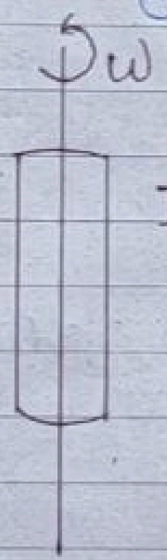
uniform rod - $\lambda = \frac{m}{L}$

a) 

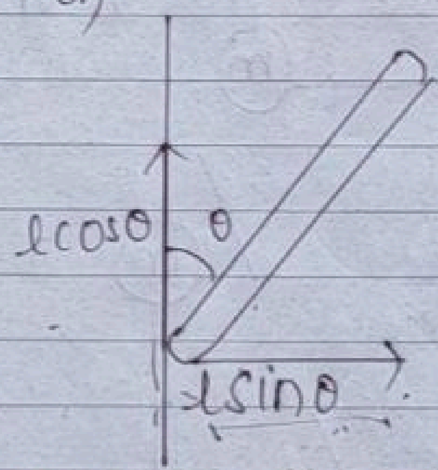
$I = \int r^2 \, dm$

$I = \int r^2 \lambda \, dr$

$I = \frac{1}{3} m L^2$

a) 

$I = 0$

a) 

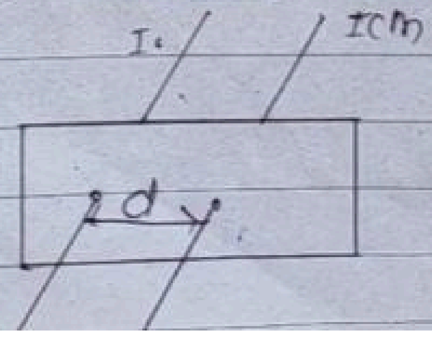
$I = 0 + \frac{1}{3} m (l \sin\theta)^2$

Parallel axis Theorem-

- axis are parallel to each other
- one axis must passing through com

$$\boxed{I_0 = I_{cm} + Md^2}$$

d - distance b/w axis



$[I_0 > I_{cm}]$

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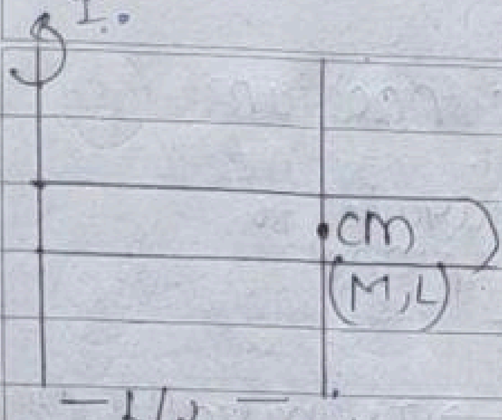
(1) I_0



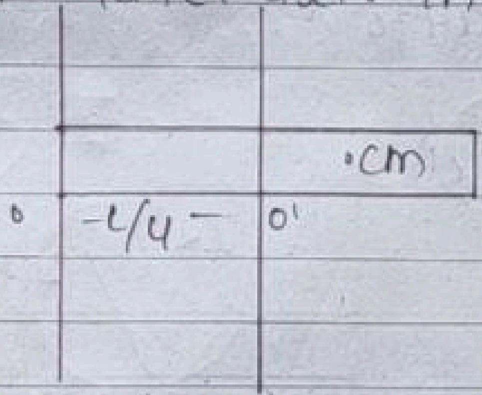
$I_0 = I_{cm} + md^2$

$\frac{ml^2}{3} = I_{cm} + \frac{ml^2}{4}$

$= \frac{ml^2}{3} - \frac{ml^2}{4}$

$I_{cm} = \frac{(4-3)Ml^2}{12} = \frac{Ml^2}{12}$

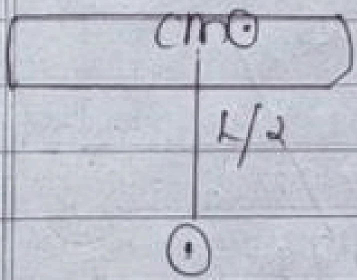
using
Parallel axis Theorum

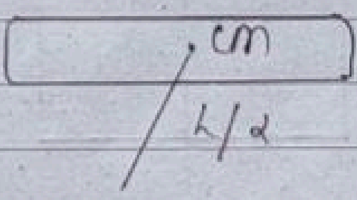


$I_0' = I_{cm} + md^2$

$\left(\frac{ml^2}{12}\right) + \frac{ml^2}{16}$

$\frac{(4+3)ml^2}{48} = \frac{7ml^2}{48}$



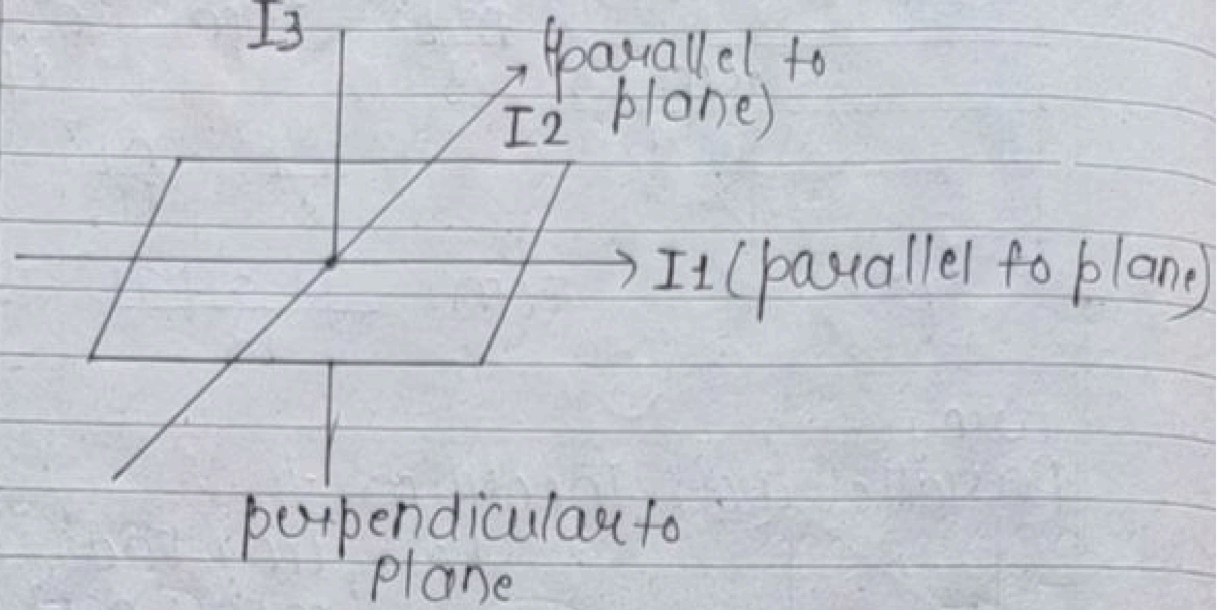


In both cases using the axis theorum.

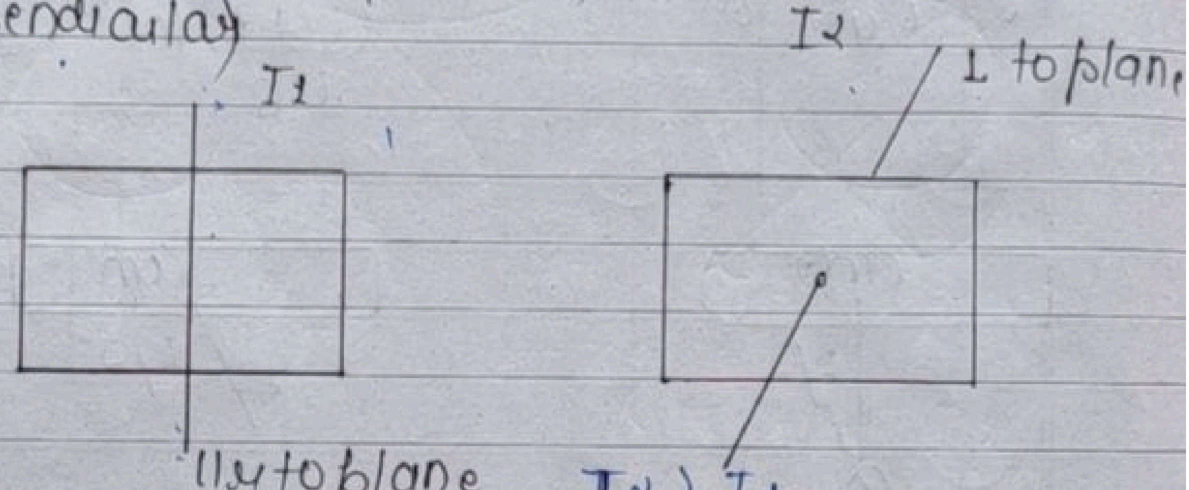
$I_0 = \frac{ML^2}{3}$

Parallel axis theorum Valid for all object

Perpendicular axis theorum.

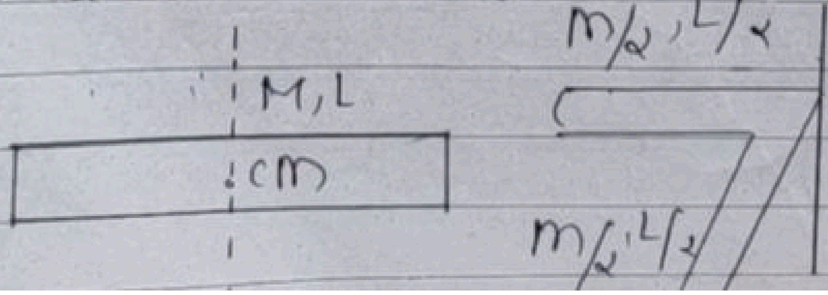


$I_3 = I_1 + I_2$ (parallel)
(perpendicular)



$I_2 > I_1$

Q) A thin rod of length L and mass M is bent at its mid point into two Halves so that the angle between them is 90°. The moment of Inertia of the bent rod about an axis passing through the bending point and perpendicular to the plane defined by two Halves of the rod is.



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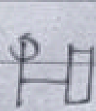
moment of Inertia of a rod from its com $\frac{ML^2}{12}$

It remain same in both cases.

Q) Rod

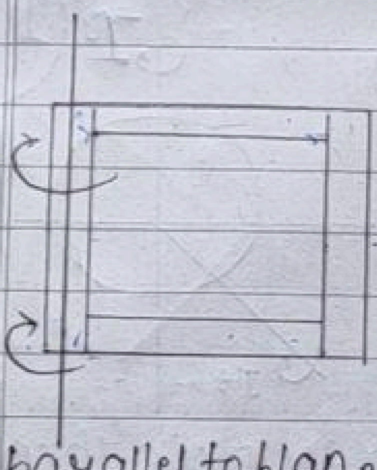


from end $= \frac{ML^2}{3}$

for point mass in rod case $I = ML^2$

from centre of mass $= \frac{ML^2}{12}$

Q)



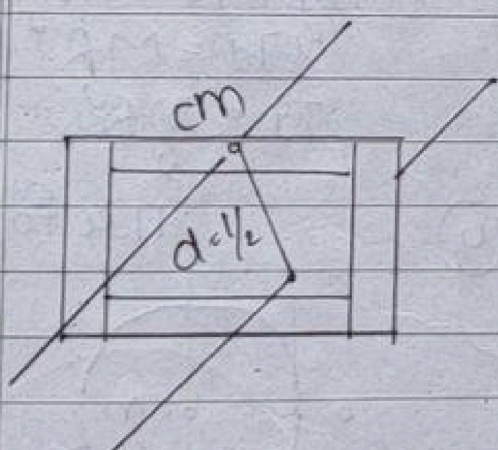
$I = I_1 + I_2 + I_3 + I_4$

$0 + \frac{ML^2}{3} + \frac{ML^2}{3} + ML^2$

$I = \frac{5ML^2}{3}$

parallel to plane

Q)

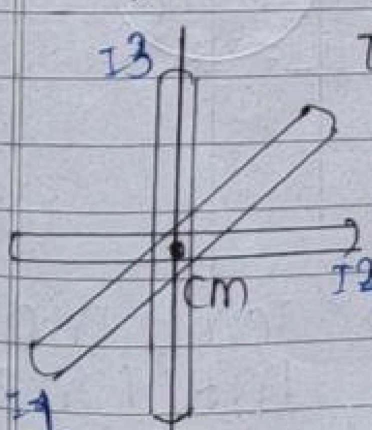


$I_0 = I_{cm} + md^2$

$= \frac{ML^2}{12} + \frac{mL^2}{4}$

$4\left[\frac{ML^2}{3}\right] =$

Q)

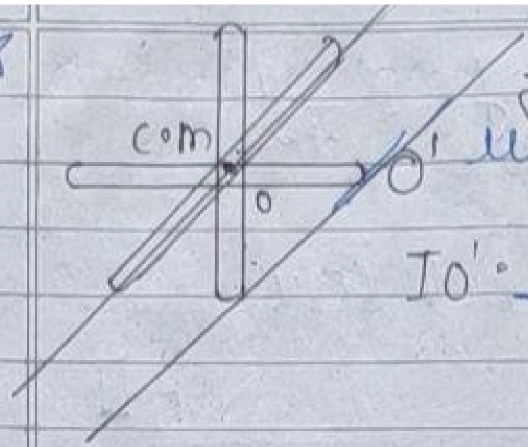


Three Identical rod ⊥ to each other MOI about y-axis

$I = I_1 + I_2 + I_3$

$\frac{mL^2}{12} + \frac{ML^2}{12} + 0 = \frac{ML^2}{6}$

☆



find $I_{O'}$

using parallel axis theorem

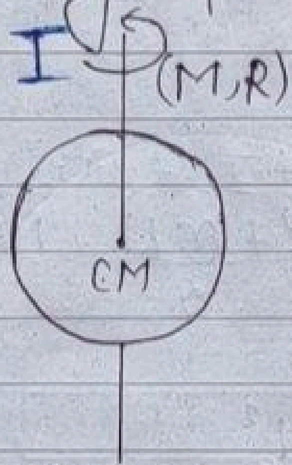
$$I_{O'} = \frac{ML^2}{6} + 3M\left(\frac{L}{2}\right)^2$$

$$\frac{ML^2}{6} + \frac{3ML^2}{4} = \frac{4ML^2 + 18ML^2}{24}$$

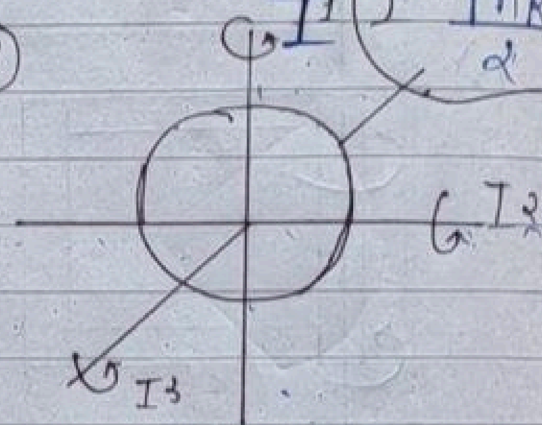
$$= \frac{22ML^2}{24}$$

Ring - [perpendicular to plane] [Parallel]

(1) $I = MR^2$



(2) $I = \frac{MR^2}{2}$



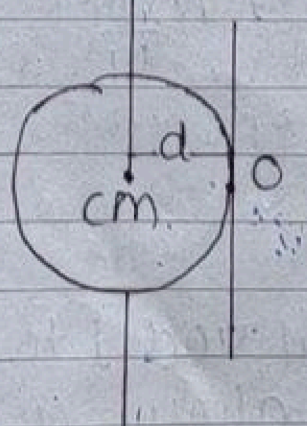
using ⊥r axis theorem

$$I_1 + I_2 = I_3$$

$$2I_1 = MR^2$$

$$I_1 = \frac{MR^2}{2}$$

(3) ⊥ to plane

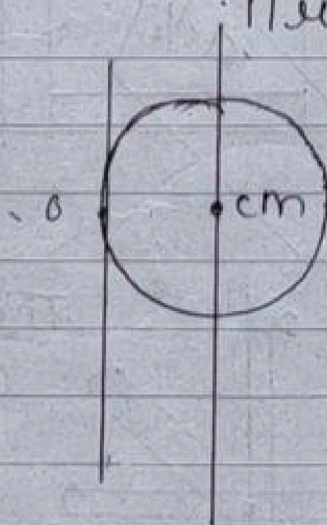


$$I_O = I_{cm} + md^2$$

$$mR^2 + mR^2$$

$$= 2mR^2$$

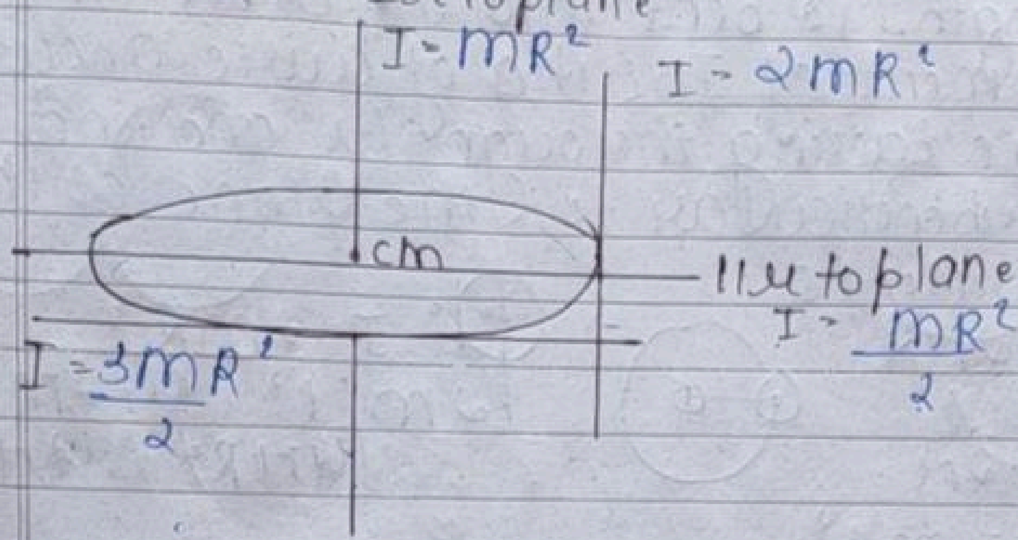
(4) ∥r to plane



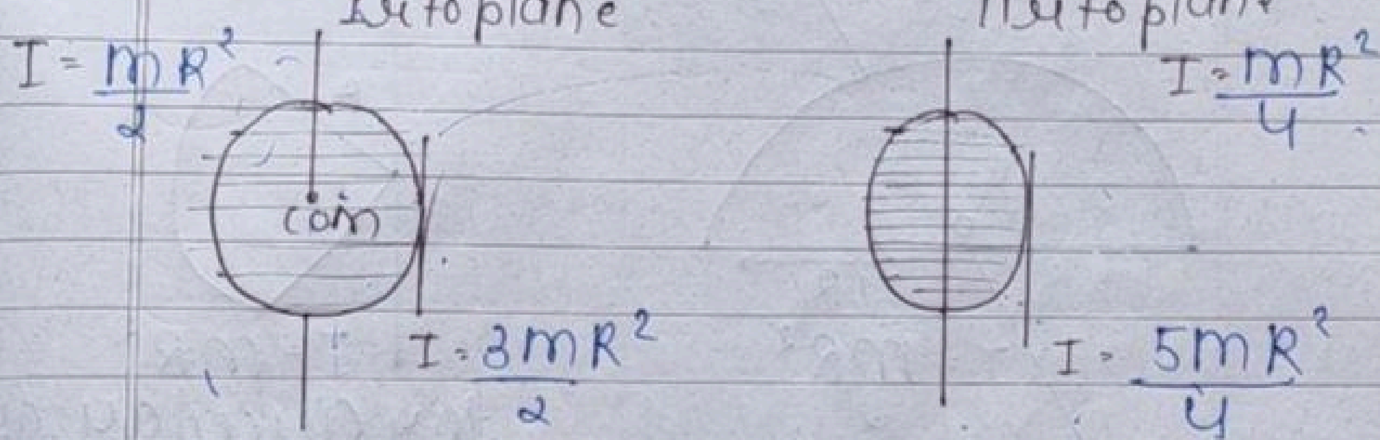
$$I_O = I_{cm} + md^2$$

$$\frac{mR^2}{2} + mR^2$$

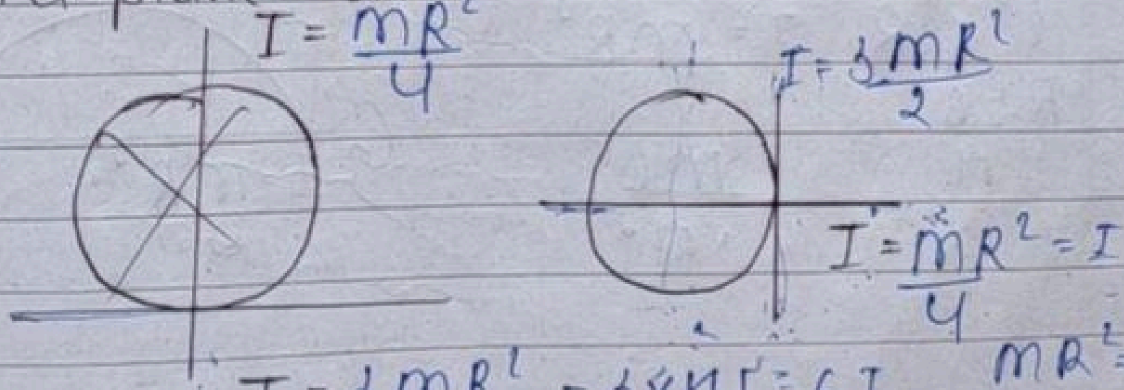
$I_0 = \frac{3MR^2}{2}$



Disc



Q) The moment of Inertia of a thin uniform circular disc about one of its diameter is I. Its moment of Inertia about an axis tangent to it and perpendicular to its plane is:

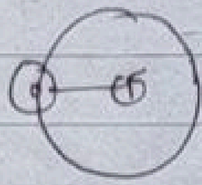


$I = \frac{3MR^2}{2} = \frac{3 \times 4I}{2} = 6I$ $\quad MR^2 = 4I$

Q) A thin uniform wire of mass m and length l is bent into a circle. The moment of Inertia of wire about an axis passing through its one end and perpendicular to the plane of the circle is.

$l = 2\pi R$

$R = \frac{l}{2\pi}$

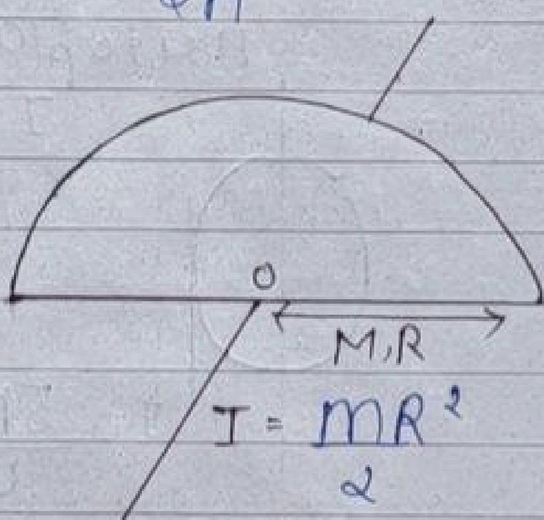


$\Rightarrow$ 

$I = 2mR^2$

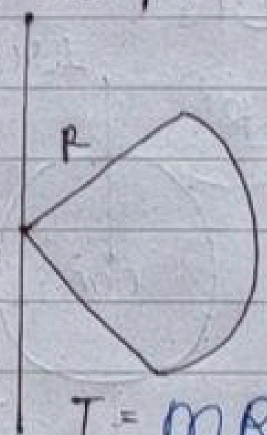
$2m\left(\frac{l^2}{4\pi^2}\right) = \frac{2ml^2}{4\pi^2}$

$I = \frac{ml^2}{2\pi^2}$



$I = \frac{MR^2}{2}$

⊥r plane



$I = MR^2$

for circular arc

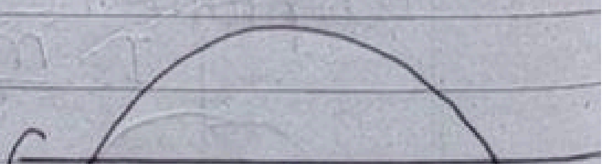
Q) A thin wire of length l and mass m is bent in the form of a semicircle as shown. Its moment of Inertia about an axis joining its free end will be.

$l = \pi R$

$R = \frac{l}{\pi}$

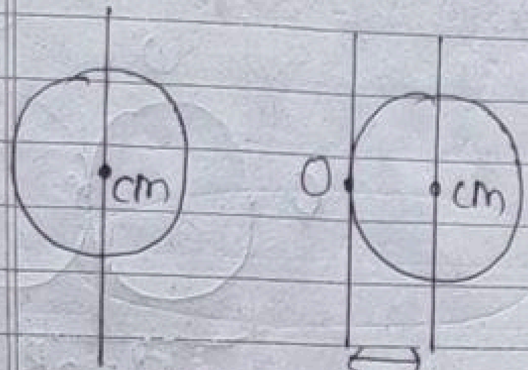
$I = \frac{mR^2}{2}$

$\frac{ml^2}{2\pi^2}$



$I = \frac{mR^2}{2}$

Hollow Sphere



$d = R$

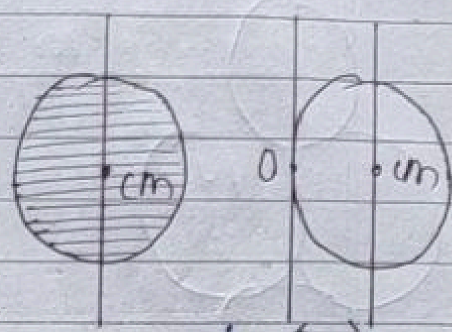
$I_{cm} = \frac{2}{3} mR^2$

$I_O = I_{cm} + mR^2$

$\frac{2}{3} mR^2 + mR^2$

$I_O = \frac{5}{3} MR^2$

Solid Sphere

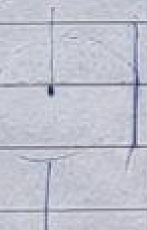


$d = R$

$I_{cm} = \frac{2}{5} mR^2$

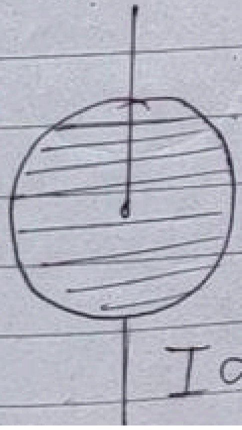
$I_O = \frac{7}{5} mR^2$

Q) The moment of Inertia of a uniform circula disc of radius R and mass M about an axis passing from the edge of the disc and normal to the disc is



$\frac{3}{2} mR^2$

Q) Two disc of same mass and same thickness have densities as 17 g/cm³ and 51 g/c the ratio of their moment of Inertia about their Central axis is
$\underbrace{\text{Central}}_{\text{passing through}}$

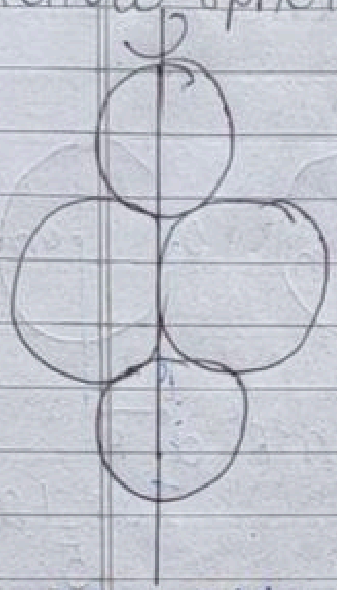


$V = \pi R^2 \times d$

$I = \frac{m(R^2 \pi d)}{2\pi d} = \frac{mV}{2\pi d} = \frac{m \times m}{(2\pi d)\rho}$

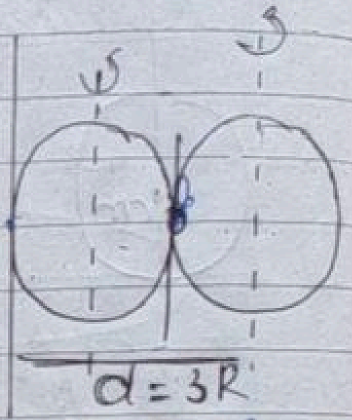
$I \propto \frac{1}{\rho} = \frac{I_1}{I_2} = \frac{\rho_2}{\rho_1} = \frac{51}{17} \quad 3:1$

4-Hollow sphere



$I = 2\left[\frac{2}{3}MR^2\right] + 2\left[\frac{5}{3}MR^2\right]$

2-Hollow sphere



$I = \frac{5}{3}MR^2 + \frac{2}{3}MR^2 + MR$

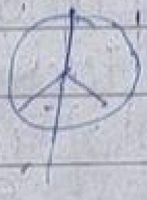
From a circular ring of mass M and radius R an arc corresponding to a 90° sector is removed. The moment of Inertia of the remaining part of the ring about an axis passing through the centre of the ring and ⊥ to the plane of ring is K times MR^2 then value of K

Initially = $I = MR^2$ $K = 3/4$

mass left = $\frac{3}{4}M$

$I = \frac{3}{4}MR^2$

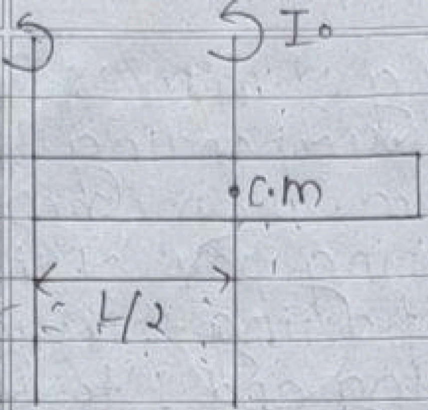
A wheel of comprises of a ring of radius R and mass M and three spokes of mass m each. The moment of Inertia of wheel about its axis is (⊥ to p)



$I = MR^2 + \left(\frac{1}{3}mR^2\right) \times 3$

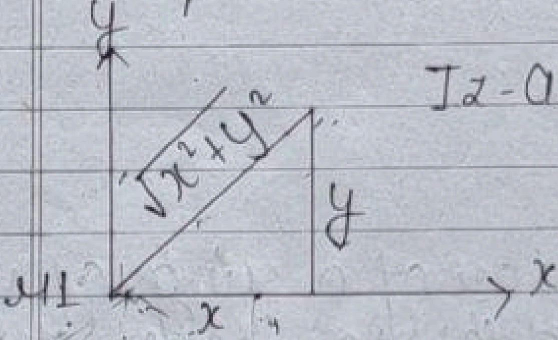
$I = MR^2 + mR^2 = (M+m)R^2$

The moment of Inertia of a thin uniform rod of mass M and length L about an axis passing through its midpoint and perpendicular to its length is I_0 its moment of Inertia about an axis passing through one of its end and ⊥ to its length is.



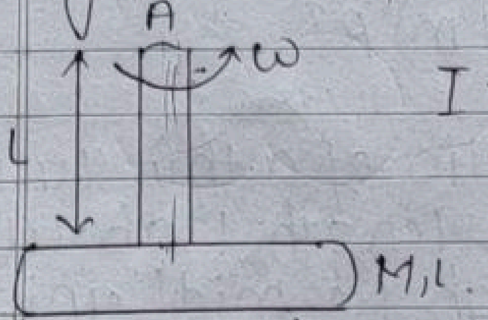
$$I_{cm} = I_0 + \frac{ML^2}{4}$$

Q) A particle of mass 1 kg is kept at (1m, 1m, 1m). The moment of Inertia of this particle about z-axis would be



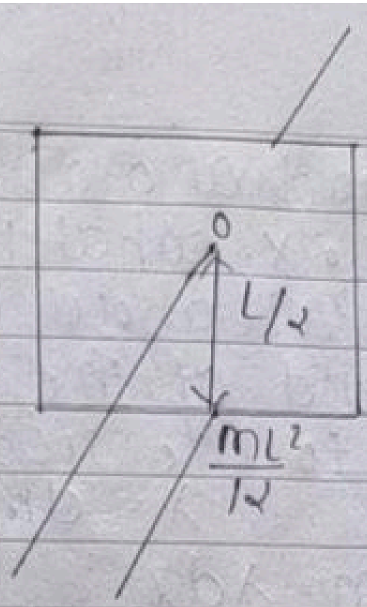
$$I_{z\text{-axis}} = m\left(\sqrt{x^2+y^2}\right)^2$$
$$= 1\left(\sqrt{(1)^2+(1)^2}\right)^2$$
$$1(\sqrt{2})^2$$
$$2\,kg$$

Q) A T joint is formed by two Identical rods A and B each of mass m and length L in xy plane. Its moment of Inertia about axis passing through end of rod ⊥ to the plane of joint.



$$I = \frac{ML^2}{3} + \left(\frac{ML^2}{12} + mL^2\right)$$
$$= \frac{17mL^2}{12}$$

Q) Four Identical thin rod each of mass M and length l from a square frame. MoI of this frame about an axis through the centre of the square and ⊥ to its plane is



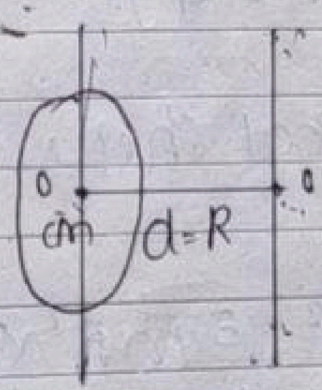
$$I = 4[I_{cm} + md^2]$$

$$4\left[\frac{ML^2}{12} + \frac{mL^2}{4}\right]$$

$$= 4\left(\frac{ML^2}{3}\right)$$

Note: jiska MOI jyada uska workdone jyada I↑ ω↑

Q) Find moment of Inertia of the ring about axis AB



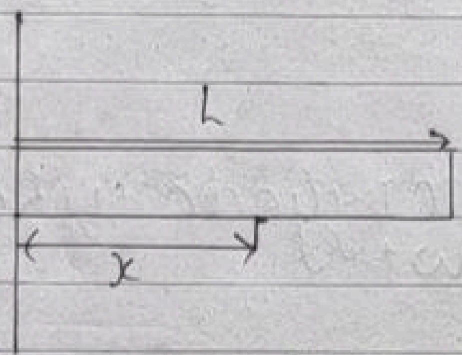
$$I_O = I_{cm} + md^2$$

$$\frac{MR^2}{2} + mR^2 = \frac{3mR^2}{2}$$

Q) Four spheres of diameter 2a and mass M are placed with centre on the the four corners of a square of side b. The MOI of the system about an axis about of the sides of square

Q.) The mass density of a rod at a distance x from its end is $\frac{m_0 x}{2L^2}$. What is the MOI of rod of length L about an axis passing through the end of the rod and perpendicular to the rod.

$\lambda = dm/dx$



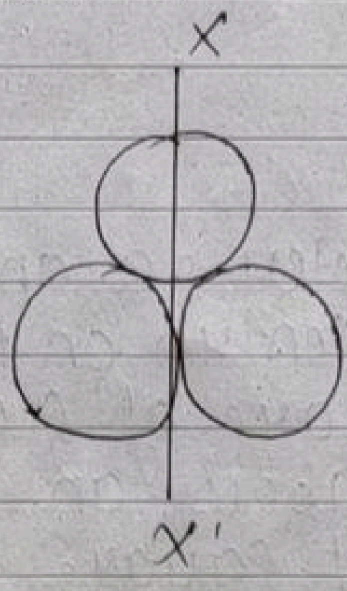
$dm = \lambda dx$

$dm = \frac{m_0}{2L^2} x\, dx$

$dI = \int x^2 dm = \int x^2 \frac{m_0}{2L^2} x\, dx$

$I = \frac{m_0}{2L^2} \int x^2 \times x\, dx = \frac{m_0}{2L^2} \left[\frac{x^4}{4}\right]_0^L = \frac{m_0 L^2}{8}$

Q.) Three identical spherical shells each of mass m.



$I = \frac{2}{3} mR^2 + \frac{5}{3} mR^2 \times 2$

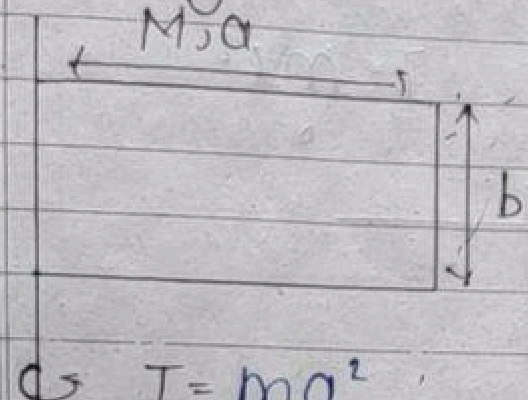
$\frac{12 mr^2}{3} = 4mr^2$

* Perpendicular axis theorem valid for planer object only disc, ring, cube, plate

Date:
Page:

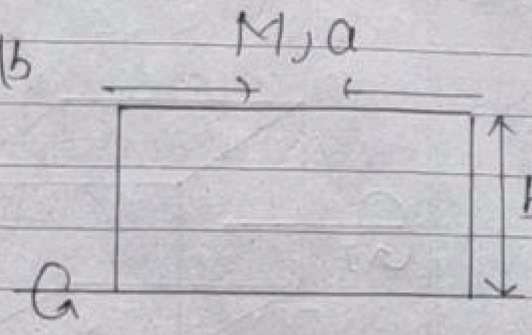
(I) Rectangular Plate

(a)

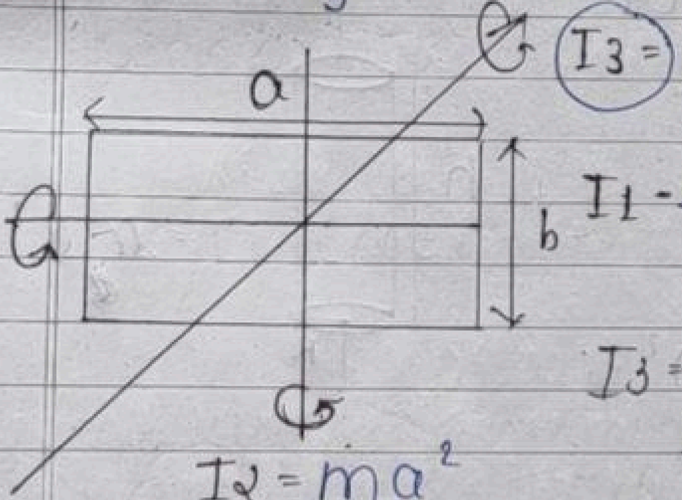


$I = \frac{ma^2}{3}$

(b)



$I = \frac{mb^2}{3}$



$I_3 =$

$I_1 = \frac{mb^2}{12}$

$I_2 = \frac{ma^2}{12}$

$I_3 = I_1 + I_2$

$\frac{ma^2}{12} + \frac{ma^2}{12}$

$I_3 = \frac{m(a^2+b^2)}{6}$

If this is a square plate

$a = b = l$

$I_3 = \frac{ml^2}{6}$

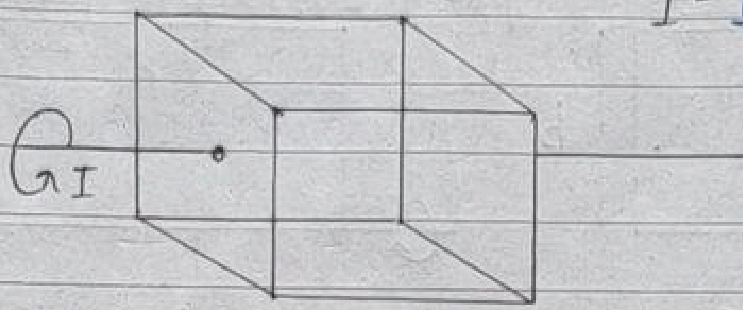
(II) Square plate

$I_1 = \frac{ml^2}{12}$

$I_3 = \frac{ml^2}{6}$

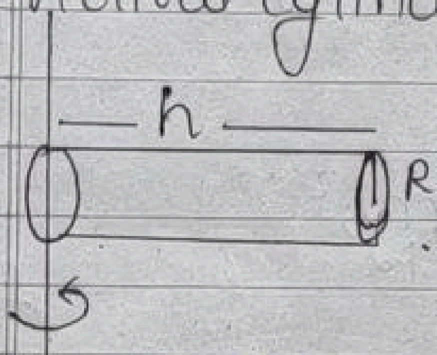
$I_2 = \frac{ml^2}{12}$

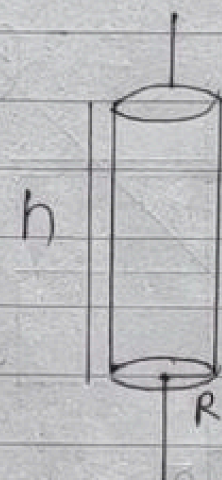
(III) Solid Cube



$$I = \frac{ml^2}{6}$$

(IV) Hollow cylinder

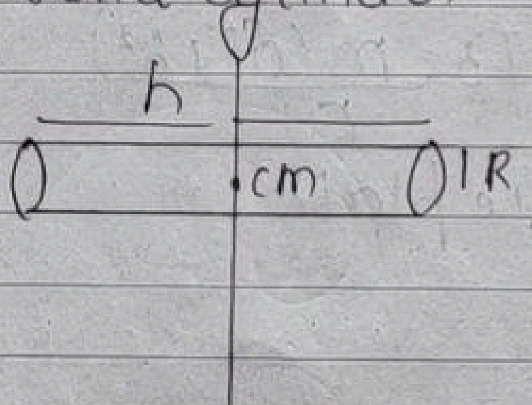


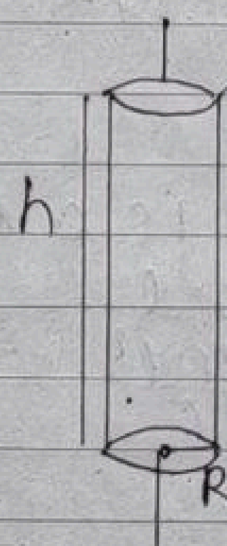


$$I = \frac{mR^2}{4}$$

$$I = mR^2 + \frac{1}{3} mh^2$$

Solid cylinder

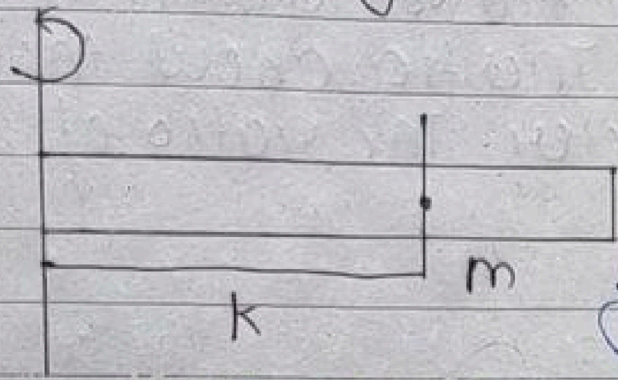




$$I = \frac{mR^2}{2}$$

$$I = \frac{mR^2}{2} + \frac{mh^2}{12}$$

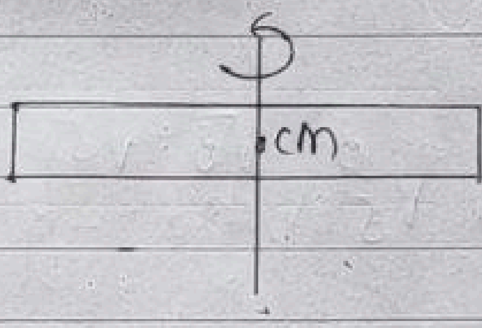
Radius of Gyration :



K - radius of gyration
a point where whole mass can be Considered for calculation of MOI

$I = MK^2 = \frac{ML^2}{3}$ $K^2 = \frac{L^2}{3}$

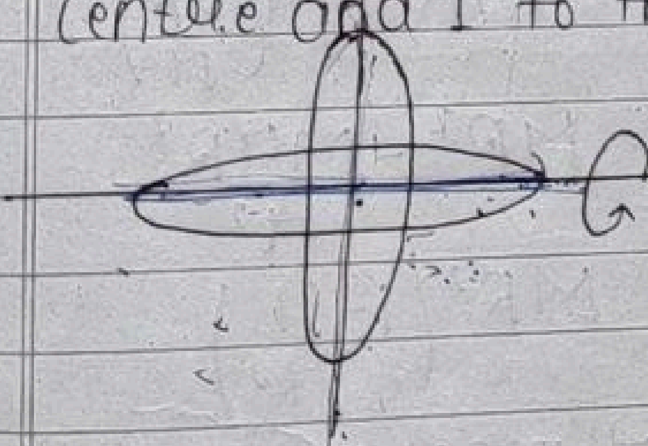
Q)



$\frac{MR^2}{12} = mK^2$

$K = \frac{R}{\sqrt{12}}$

Q) Two rings of same mass and radius R are placed with their planes ⊥ to each other and centres at a common point The radius of gyration of the system about an axis passing through the centre and ⊥ to the plane of one ring is



$I = MR^2 + \frac{mR^2}{2} = \frac{3mR^2}{2}$

$2\cancel{m}K^2 = \frac{3\cancel{m}R^2}{2}$

$K^2 = \frac{3R^2}{4}$ $K = \frac{\sqrt{3}R}{2}$

$I_{cm} + md$

Date:
Page:

Q) The two spheres one of which is Hollow and other Solid, Have Identical mass and moment of Inertia about their respective diameter. The ratio of their radii given by

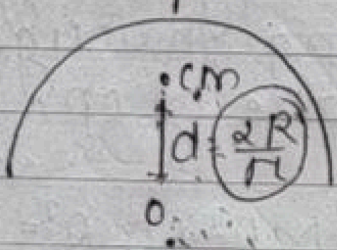
$$I_1 = I_2$$

$$\frac{2}{3}\cancel{m} r_1^2 = \frac{2}{5}\cancel{m} R_2^2$$

$$\frac{1}{3} r_1^2 = \frac{1}{5} r_2^2$$

$$\frac{r_1^2}{r_2^2} = \frac{3}{5} \quad \frac{r_1}{r_2} = \sqrt{\frac{3}{5}} \quad \sqrt{3} : \sqrt{5}$$

Q.) The moment of Inertia of a uniform semicircular wire of mass m and radius r, about an axis passing through its com and perpendicular to its plane is



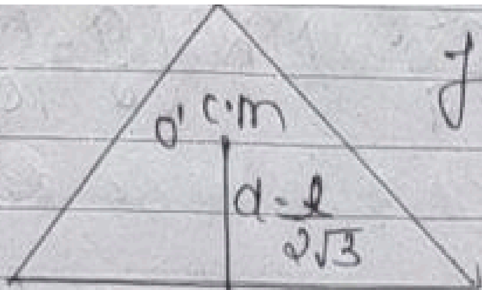
$$I_O = I_{cm} + md^2$$

$$MR^2 = I_{cm} + M\left(\frac{2R}{\pi}\right)^2$$

$$I_{cm} = MR^2 - \frac{M4R^2}{\pi^2}$$

$$I_{cm} = MR^2\left(1 - \frac{4}{\pi^2}\right)$$

Q) Three rods each of mass m and length l are joined to form equilateral Δ what is MOI about an axis passing through the com of system and ⊥ of plane.



for one rod

$I_{O'} = I_O + md^2$

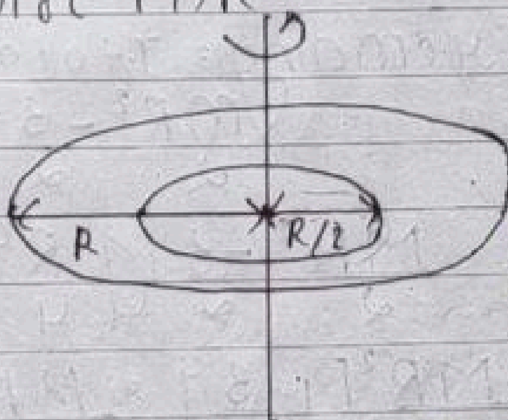
$\frac{ml^2}{12} + m\left(\frac{l}{2\sqrt{3}}\right)^2$

$I_O = \frac{ml^2}{12}$ $= ml^2\left[\frac{1}{12} + \frac{1}{12}\right] = \frac{ml^2}{3}$

$I_{system} = I_{rod} \times 3$

$\frac{Ml^2 \times \cancel{3}}{\cancel{6}\,2} = \frac{Ml^2}{2}$

Q) Disc M, R:



Disc of radius R/2 is removed then find MOI of remaining

πr^2 M

1 unit $\frac{M}{\pi r^2}$

$\pi\left(\frac{R}{2}\right)^2 = \frac{M}{\pi r^2} \times \frac{\pi R^2}{4} = M/4$

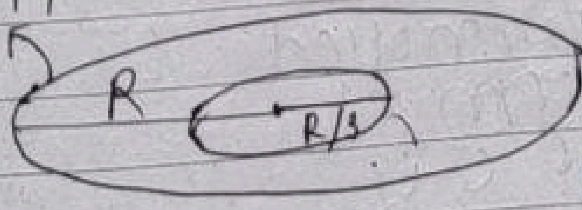
$I_{remaining\ part} = I_{total} - I_{removed}$

$\frac{MR^2}{2} - \frac{M'R'^2}{2}$

$\frac{MR^2}{2} - \frac{M}{4} \times \frac{R^2}{4} \times \frac{1}{2} =$

$\frac{MR^2}{2}\left[1 - \frac{1}{16}\right] = \frac{15M^2}{32}$

Q) R, 9M



πr^2 9M

1 unit $\frac{9m}{\pi r^2}$

$\pi\frac{R^2}{9} = \frac{9m}{\pi r^2} \times \frac{\pi R^2}{9} = M$

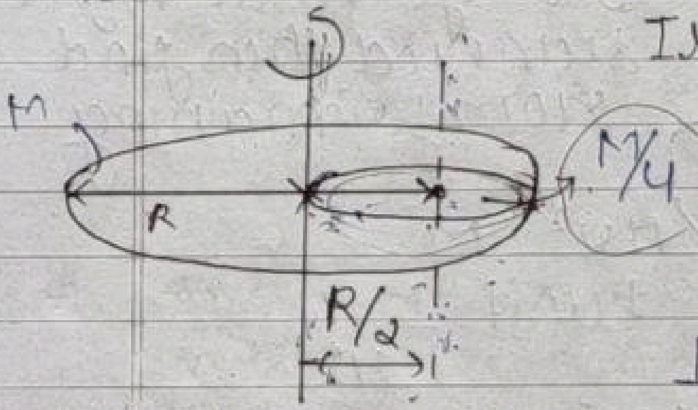
Date:
Page:

$I_{remaining} = I_{total} - I_{removed}$

$$\frac{9MR^2}{2} - \frac{MR^2}{2\times 9} = MR^2\left[\frac{9}{2} - \frac{1}{18}\right] = \frac{80}{18}$$

$$\frac{40}{9}MR^2$$

Q) from a disc of radius R and mass M a circular Hole of diameter R, whose rim passes through the centre is cut. what is the moment of Inertia of the remaining part of Disc about a ⊥ axis, passing through centre?



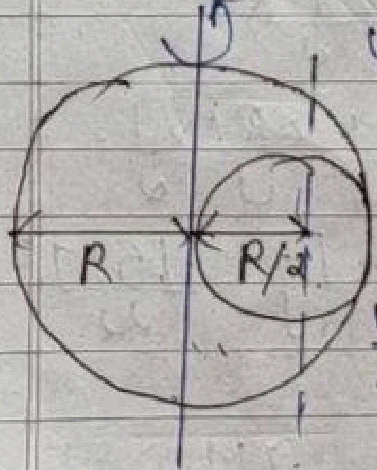
$I_{remaing} = I_{total} - I_{re}$

$$= \frac{MR^2}{2} - \frac{3}{2}mR^2$$

$$\frac{MR^2}{2} - \frac{3}{2}\frac{M}{4}\frac{R^2}{4}$$

$$\frac{MR^2}{2}\left[1 - \frac{3}{16}\right] = \frac{13MR^2}{32}$$

Q) A solid sphere (M, R) and solid sphere of radius (R/2) is removed.



$\frac{4}{3}\pi r^3 \quad M$

$1 \text{ unit } \frac{m}{\frac{4}{3}\pi r^3}$

$$\frac{4}{3}\pi\frac{r^3}{8} = \frac{m}{\cancel{\frac{4}{3}\pi r^3}} \times \frac{\cancel{\frac{4}{3}\pi r^3}}{8} = m/8$$

$I_{remaining} = I_{total} - I_{removed}$

$$\frac{2}{5}mR^2 - \frac{7}{5}\frac{m}{8}\frac{R^2}{4}$$

Date:
Page:

Torque moment of force

$\vec{\tau} = I\vec{\alpha}$ $\vec{F} = m\vec{a}$

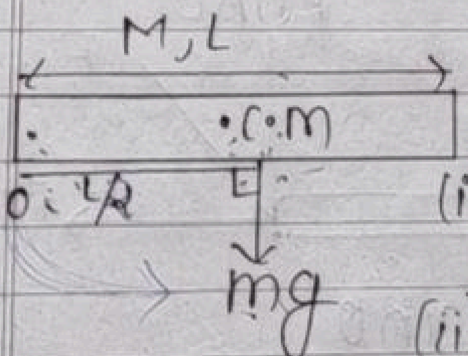
$\vec{\tau} = \frac{d\vec{\tau}}{dt}$ $f = \frac{dp}{dt}$

$\vec{\tau} = \vec{r} \times \vec{F}$ $\vec{\tau} \cdot \vec{r} = 0$

m-H. $\vec{\tau} \cdot \vec{F} = 0$

→ Causes of change in Rotational State of the Body
→ moment of force
→ Turning effect Force, Distance, Directi
→ Vector
→ unit : [N-m] $[ML^2T^{-2}]$

Q) Find torque due to gravitational force on rod?



(i) about com
$\tau = 0$

(ii) about 'O'
$\tau = mg\frac{L}{2}$

Direction of torque
$\tau = \vec{r} \times \vec{F}$

thumb will represent torque
place your four fingure along r
flap force

★★ $\tau_{net} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 \ldots$

(1) If $F_{net} = 0$, then τ_{net} must be zero F

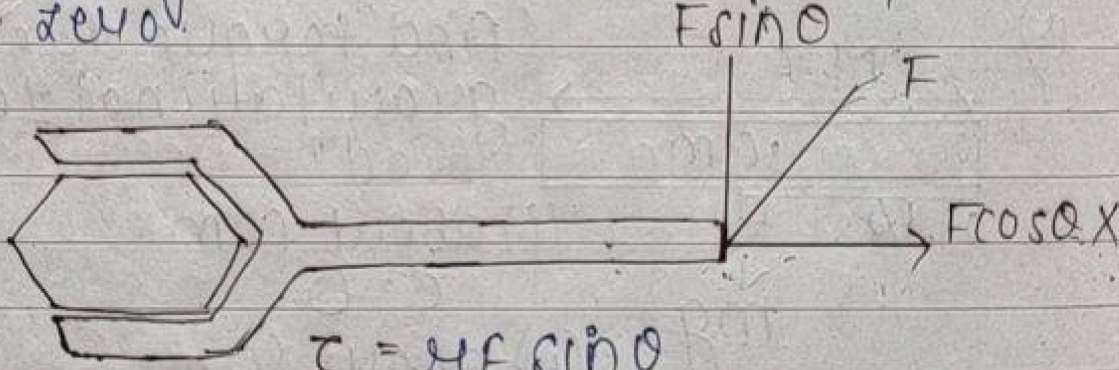
(2) If $F_{net} \neq 0$ then τ_{net} must be zero F

(3) If $\tau_{net} = 0$ then F_{net} must be non-zero F

(4) If $\tau_{net} \neq 0$ then F_{net} must be non-zero F

(5) If no force is acting on the body. then τ_{net} must be zero True

⇒ If line of force is passing through axis of rotation then torque will be zero.



$\tau = rF\sin\theta$

$\tau = \vec{r} \times \vec{F}$

The moment of the force $\vec{F} = 4\hat{i} + 5\hat{j} - 6\hat{k}$ at $(2, 0, -3)$, about the point $(2, -2, -2)$ given by

$r \rightarrow$ position vector of force

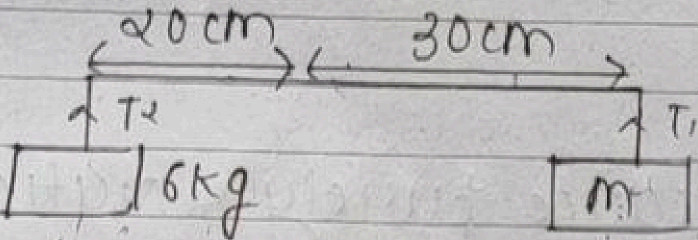
$\vec{r} = 2\hat{i} - 3\hat{k} - (2\hat{i} + 2\hat{j} + 2\hat{k})$

$0+2\hat{j}-\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & -1 \\ +4 & +5 & -6 \end{vmatrix} \qquad \begin{aligned} &\hat{i}(-12+5) \\ &-\hat{j}(0-(-4)) \\ &+\hat{k}(-8) \end{aligned}$$

$-7\hat{i}-4\hat{j}-8\hat{k}$

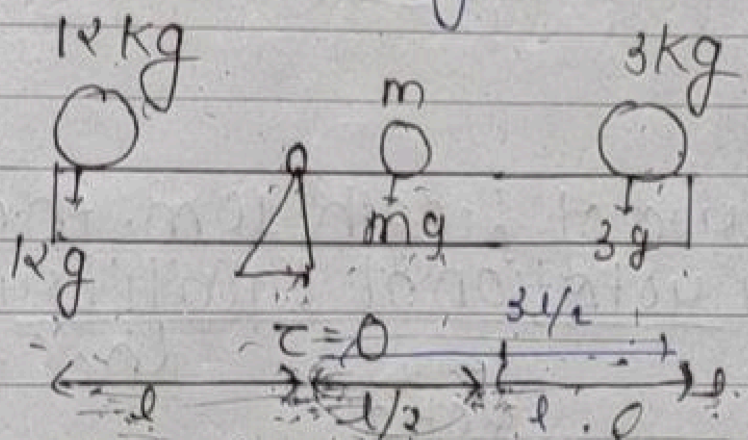
8)



$\cancel{60}\times 2\cancel{0} = mg\times 3\cancel{0}$

$m = 4\,kg$

9)



Distance main point (balance)

$12gl = mg\frac{l}{2} + 3g\times\frac{3l}{2}$

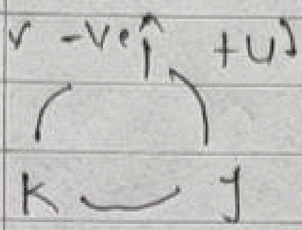
$2\times 12 = \frac{m}{\cancel{2}} + \frac{9}{\cancel{2}} \qquad 24-9=m$

$m = 15\,kg$

$\Sigma F_{net} = 0$ Translational equilibrium

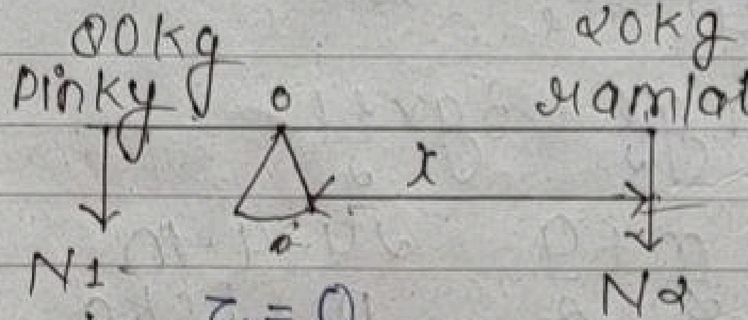
$\Sigma \tau_{net} = 0$ Rotational equilibrium

Mechanical Disadvantage $= \dfrac{F_{large}}{F_{small}} = \dfrac{d_{large}}{d_{small}}$



A couple produce purely rotational motion.
couple : Two equal and opposite force acting at same distance
$F_{net} = 0$
$\tau_{net} \neq 0$

Q) A massless rod of length 10m. then find x for rotational equilibrium about O. $\tau_{net} = 0$



$\tau_O = 0$

$80g(10-x) = 20gx$

$x = 4(10-x)$

$5x = 40$

$x = 8m$

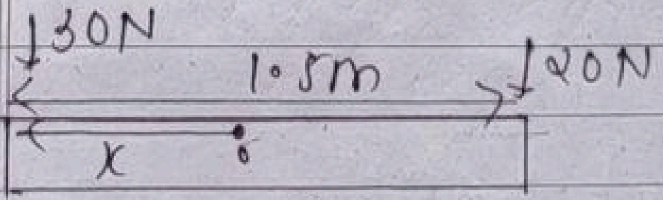
Q) The angle between Vector $(\vec{M} \times \vec{N})$ and $(\vec{N} \times \vec{M})$

180°

Q) moment of a force of magnitude 20N acting along the x- direction at point (3m, 0, 0) about the point (0, 2, 0) in N-m

$\vec{r} = 3 + 0 + 0 - (0 + 2 + 0)$
$= 3\hat{i} + 2\hat{j} + 0\hat{k}$

$\vec{\tau} = r \times F = (3\hat{i} - 2\hat{j}) \times 20\hat{i}$
$= +40\hat{k}$

Q) 

$\tau_0 = 0$
$30x = 20(1.5 - x)$
$3x = 3 - 2x$
$5x = 3$
$x = \frac{300}{5} = 60\text{cm}$

Total = 150cm
from A = 90cm

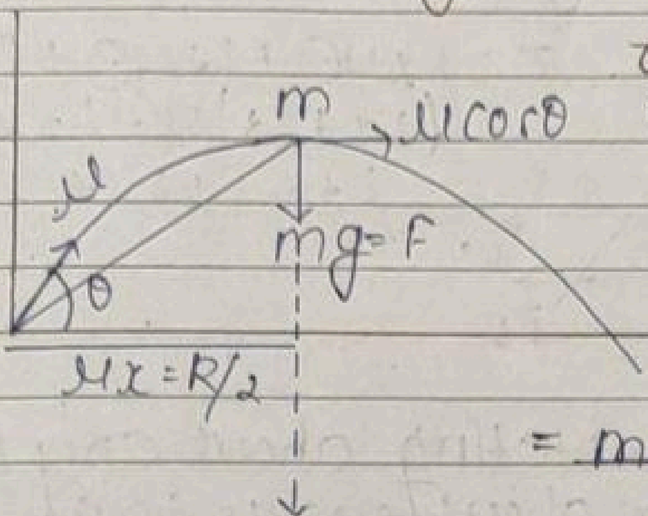
Q) Vector $\vec{A}$ points towards North and Vector $\vec{B}$ points towa. upward then $\vec{A} \times B$ points toward

Right Hand $\vec{A}$ ki taraf thappad B k. direction thum

west

Date
Page

Q) A particle of mass m has been thrown up with Initial speed u making angle θ with the Horizontal. Find the torque of its weight about the point of projection when it just reached the Highest point.



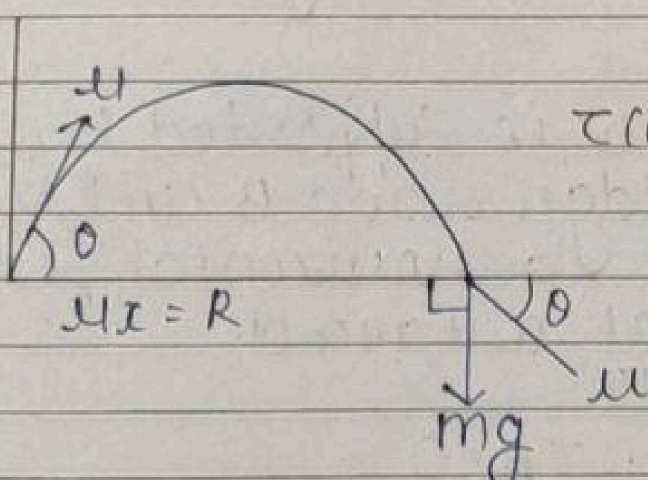
τ due to mg about (0,0) $= mg(y) \times \left(\frac{R}{2}\right)$

$= \frac{mgR}{2}$

$= \frac{mg\,u^2 \sin(2\theta)}{2g}$

$= \frac{mu^2 \sin(2\theta)}{2}$

(II)

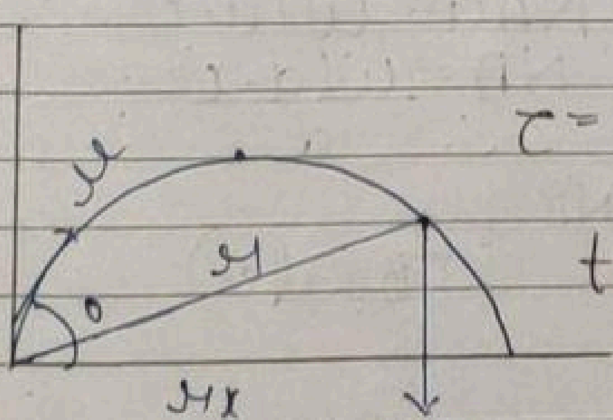


$\tau(0,0) = mgR$

$\frac{mg\,u^2 \sin(2\theta)}{g}$

$= mu^2 \sin(2\theta)$

(III)

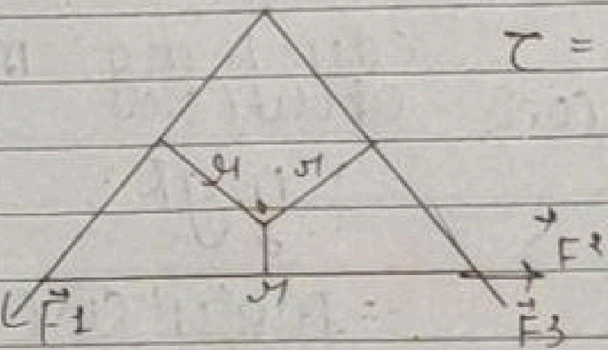


$\tau = mg \times u\cos\theta\, t$

$t = \frac{u\sin\theta}{g}$

$r = u\cos\theta \times t$

ABC is an equilateral triangle with O as its centre F_1, F_2 and F_3 represents 3 forces acting along the sides AB, BC and AC. if the total torque about O is zero then the magnitude of F_3 is.



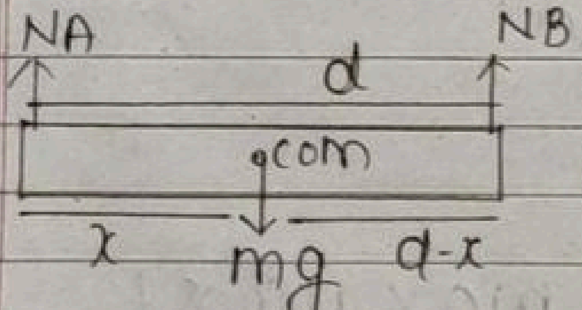
$$\tau = F_1 r(\odot) + F_2 r(\odot) + F_3 r(\otimes) = 0$$

$$(F_1 + F_2) r = F_3 r$$

$$F_3 = F_1 + F_2$$

If body isn't rotating about any point then torque about every point will be zero.

Q) A rod of weight W is supported by two parallel knife edges A and B and it is equilibrium in a Horizontal position The knives are at distance d.

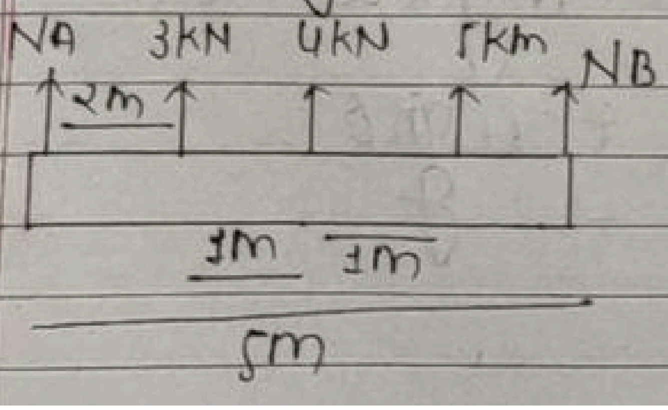


$$\tau B = 0$$

$$N_A d = W(d-x)$$

$$N_A = \frac{W(d-x)}{d}$$

Q)

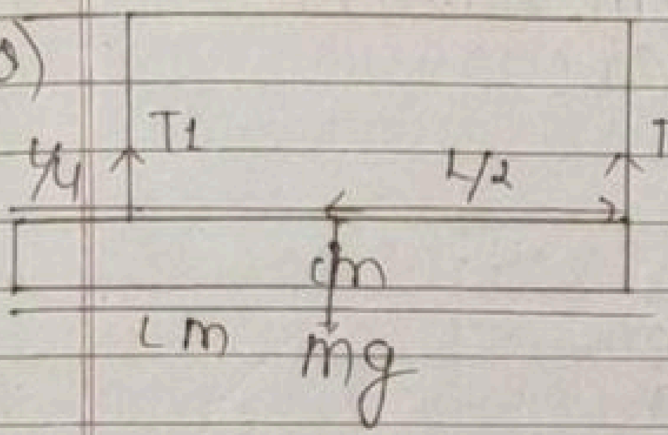


$$\tau B = 0 \; (ct)$$

$5N_A (+) = 3kN \times 3 + 4kN \times 2 + 5kN \times 1$

$N_A = \frac{22kN}{5}$

a)

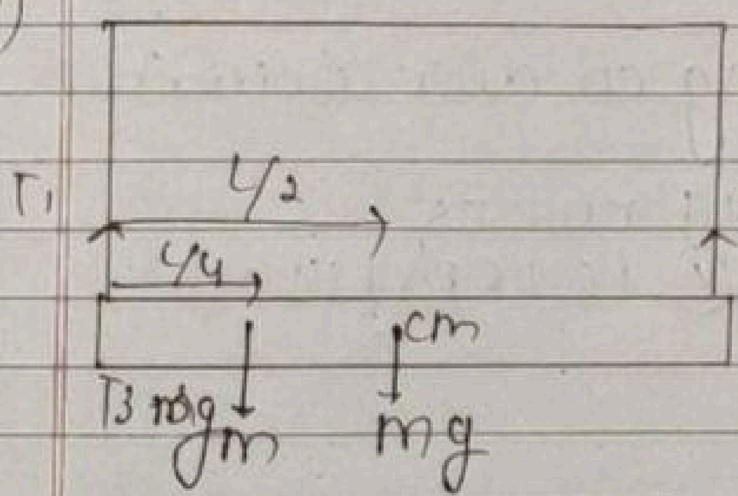


$(\tau_{net})_2 = 0$

$T_2 + mg\frac{L}{2} = T_1\frac{3L}{4}$

$T_1 = \frac{2mg}{3}$

a)



for T_2

$(\tau_{net})_{T_1} = 0$

$mg\frac{L}{2} + mg\frac{L}{4} = T_2 L$

$T_2 = \frac{mg}{4} + \frac{mg}{2}$

Date
Page

Q) Find torque about 'O' due to gravitational force when rod at angle 'θ' from vertical

(τ_{mg}) about c.o.m. O

$= (mg)_{vertical} \times (r_\perp)_x$

$mg \frac{L}{2} \sin\theta$

$= \frac{mgL\sin\theta}{2}$

If $\theta = 90^\circ$, $\tau = \frac{mgL}{2}$ If $\theta = 0^\circ$, $\tau = 0$

Q) Find torque acting on disc about O.

$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4 + \tau_5$

$FR \otimes + \frac{F}{2} R \otimes + 0 + FR \times \frac{1}{2} \cos 60^\circ$

$FR \otimes$

Rotation-

Pure Translation motion- If every part of particle is moving with same translational velocity. Then object is in pure T. motion

Pure rotational motion-

If every particle of the body moves is a circle & centre of all the circles lies on the same straight line called axis of rotation.

Date
Page

every point of Body is moving with different Velocity but same angular speed about fixed axis of rotation.

Rolling motion: Combined Translational and Rotational motion is called rolling motion

Circular Motion	Rotational Motion
→ when object moves on the circumference of the circle object → Point	all the particle of rigid body move on circular path of diff radius, with common centre.

Angular Velocity Same for each point, but different linear Velocity.

$\omega_A = \omega_B = \omega_C$

$v_1 \neq v_2 \neq v_3$

Translational Motion and Rotational mot

Displacement - s (θ)

Velocity $v = \frac{ds}{dt}$ $\omega = \frac{d\theta}{dt}$

Acceleration $\vec{a} = \frac{d\vec{v}}{dt}$ $\alpha = \frac{d\omega}{dt}$

Date
Page

Force $\vec{F} = \frac{dp}{dt} = m\vec{a}$ $\tau = I\alpha = \frac{d\vec{L}}{dt}$

momentum(P) $= m\vec{v}$ $L = I\omega$

work $= Fs = \int F \cdot ds$ work $= \vec{\tau} \cdot \theta = \int \vec{\tau} d\theta$

$K.E = \frac{1}{2} m v^2 = \frac{P^2}{2m}$ $K.E = \frac{1}{2} I \omega^2 = \frac{L^2}{2I}$

Power (P) $= F \cdot v = \frac{F \cdot s}{T} = \frac{W}{T}$ $P = \vec{\tau} \cdot \vec{\omega}$

Impulse $= m \Delta v = \int F \cdot dt = \Delta P$

Angular Impulse $\Delta L = I \Delta \omega$

Equation of Motion (α = const)

$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$ $\theta = \frac{1}{2} \left(\frac{\alpha \cdot \beta}{\alpha + \beta} \right) T^2$

$\omega_f = \omega_i + \alpha t$ $\omega_{max} = \left(\frac{\alpha \cdot \beta}{\alpha + \beta} \right) T$

$\omega_f^2 - \omega_i^2 = 2\alpha\theta$

$\theta_{n^{th}} = \omega_0 + \frac{\alpha}{2}(2n-1)$

$\theta = \left(\frac{\omega_f + \omega_i}{2} \right) T$

ω_0 - Initial angular Velocity
α - angular retardation
$\theta = ?$
$\omega_f = 0$

$$\theta = \frac{\omega_0^2}{2\alpha}$$

Q) what is the angular accⁿ of a particle if the angular velocity of a particle becomes 4 times of its Initial angular Velocity 1 rad/s in 2 sec.

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{4-1}{2} = \frac{3}{2} = 1.5 \text{ rad/s}^2$$

Q) A particle moving with an angular Velocity of 200 rad s^{-1} start de-accelerating at a rate of 2 rad s^{-2}. calculate the time in which it will come to rest.

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \quad -2 = \frac{0-200}{t} \quad t = 100 \text{ sec}$$

Q) A fan is rotating with a speed of 450 rev/min. after being switched off it comes to rest in 10 s. assuming constant angular deceleration, calculate the no. of revolutions made by it before coming to rest.

$f = 450\ rev/min$ $\omega = 2\pi f = \cancel{2}\pi \frac{45}{\cancel{6}_3}$

$f = 450\ rev/s$ $= 15\pi\ rad/s.$

$f = \frac{45}{6}\ rev/s$

$\theta = \frac{15\pi + 0}{2} = 15\pi$

$\text{no. of rotation} = \frac{\theta}{2\pi} = \frac{15\cancel{\pi}}{2\cancel{\pi}} = 37.5$

Q) A disc rotating about its axis from rest acquires the angular speed 100 rev/s in 4 sec. The angle by it during these 4 sec (rad).

$\omega_i = 0$ $\theta = \left(\frac{\cancel{2}00\pi + 0}{\cancel{2}}\right)\cancel{4}^2$

$f = 100\ m/s$

$\omega_f = 2\pi f = 200\pi$ $\theta = 400\pi$

Q) A uniform rod of 20kg is hanging in a Horizontal position with the help of two threads. It also supports a 40 kg. Find the tension developed in each thread.

$\tau A = 0$

$400 \times \frac{\cancel{1}}{4} + 200 \times \frac{\cancel{1}}{2} = TB \times \cancel{1}$

$100 + 100 = TB$

$TB = 200N$

Page

$F_{net} = 0 \quad T_A + T_B = 600N$

$T_A = 400N$

Q) The Instantaneous angular position of a point on a rotating wheel is given by the equation $\theta(t) = 2t^3 - 6t^2$. the torque on the wheel becomes zero at.

$\tau = 0$

$a = 0$

$\omega = \frac{d\theta}{dt} = 6t^2 - 12t$

$\alpha = \frac{d\omega}{dt} = 12t - 12 = 0$

$t = 1\ sec$

Q) A Body rotating with uniform angular acc^n covers 100π in first 5 s after the start. Its angular speed at the end of 5 s is

$\omega_i = 0$

$\theta = 100\pi$

$t = 5\ sec$

$\omega_f = ?$

$\theta = \left(\frac{\omega_i + \omega_f}{2}\right) T$

$\cancel{100}^{20}\pi = \left(\frac{0 + \omega_f}{2}\right) \times \cancel{5}$

$\omega_f = 40\pi$

Q) A body rotate about a fixed axis with a angular acc^n of 3 rad/s². The angle rotate by it during the time when its angular velocity Increases from 10 rad/s to 20 rad/s

Date
Page

Q.3 $\omega_f^2 - \omega_i^2 = 2\alpha\theta$

$400 - 100 = 2 \times 3\theta$

$300 = 6\theta$

$\theta = 50$ rad.

Two particles start moving from same position on a circle of radius 20 cm with speed 4π m/s and 8π m/s in the same direction. find

$V_{AB} - 4\pi$ m/s $\quad t = \frac{dist^n AB}{V_{AB}} = \frac{2\pi R}{4\pi} = \frac{1}{2}\left(\frac{20}{100}\right)$

$= 0.1$ sec

If the parallel forces acting on a lever are in the ratio $3:5$ then what is the mechanical advantage of the lever

$M.A = \frac{5}{3}$

The grinding stone of a flour mill is rotating at 600 rad/s for this power is 1.2 k watt is used. The effective torque on stone in N-m will be

$P = mv$

$P = \tau \cdot \omega$

$1.2 kW = \tau \times 600$

$12 \times 100 = \tau \times 600$

$\tau = 2$ Nm

Date
Page

Question on Circular Dynamics-

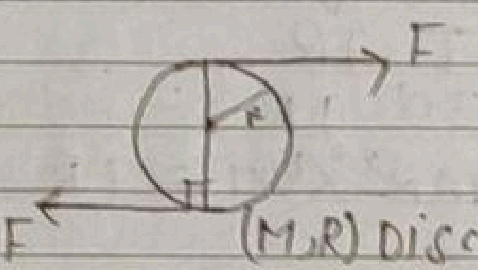
Q) Two equal and opposite forces are applied tangentially to a uniform disc of mass M and radius R. If the disc is pivoted at its centre and free to rotate in its plate, the angular accⁿ of the disc is.

$\tau_{net} = FR + FR$

$I\alpha = 2FR$

$\frac{MR^2}{2}\alpha = 2FR$

$\alpha = \frac{4F}{MR}$



A wheel having MOI $2\,kg\text{-}m^2$ about its vertical axis rotates at the rate of 60 rpm about the axis. The torque which can stop the wheel's rotation in one min. would be.

$I = 2\,kgm^2$

$f = 60\,rpm = \frac{60}{60}\,rps$

$\omega = 2\pi f = 2\pi\,rd/s$

$\omega_f = 0$

$t = 60\,sec$

$\alpha = \frac{\omega_f - \omega_i}{\Delta t}$

$\alpha = -\frac{2\pi}{60}$

$\tau = I\alpha = 2 \times \left(-\frac{2\pi}{60}\right) = -\frac{\pi}{15}$

Q) A solid cylinder of mass 50 kg and radius 0.5m is free to rotate about the Horizontal axis. a massless string is wound around the cylinder with one end attached to it other hanging freely. Tension in string required to produce an angular accn of 2 revolution s^{-2} is.

$2 rev/s^2$

$\alpha = 2\pi (2 rev/s^2)$

$= 4\pi \ rev/s^2$

$I = TR$

$I\alpha = TR$

$\frac{mR^{\cancel{2}}}{2} \times 4\pi = T\cancel{R}$

$m(2\pi)R = T$

$50 \times 2\pi \times \frac{1}{2} = T$

Q) A rope is wound around a Hollow cylinder of mass 3kg and radius 40cm. What is the angular accn of the cylinder if the rope is pulled with a force of 30N?

$\tau = TR$

$I\alpha = TR$

$MR^{\cancel{2}}\alpha = T\cancel{R}$

$\alpha = \frac{T}{mR} = \frac{\cancel{30} \times 10\cancel{0}}{\cancel{3} \times 4\cancel{0}} = 25$

Q) A wheel has angular accn of 3 rad/s^2 and an initial angular speed of 2 rad/

Date
Page

In a time of 2 sec it has rotated through an angle.

$\alpha = 3 \text{ rad}/s^2$ $\quad t = 2 \sec$

$\omega_i = 2 \text{ rad}/s$

$$\theta = \omega t + \frac{1}{2}\alpha t^2 = 2\times 2 + \frac{1}{2}\times 3(2)^2$$

$$= 4 + \frac{1}{2}\times 3\times 4 = 10$$

Q.) A solid body rotate about a fixed axis so that its angular velocity depends on θ as $\omega = k\theta^{-1}$ where k is a +ve constant. At $t = 0, \theta = 0$,

$\omega = k\theta^{-1}$

$\frac{d\theta}{dt} = \frac{k}{\theta}$

$$\int_0^\theta \theta\, d\theta = \int_0^t k\, dt$$

$$\frac{\theta^2}{2} = kt \qquad \theta = \sqrt{2kt}$$

Q) A solid cylinder of mass 2kg and radius 4cm is rotating about its axis at the rate of 3 rpm. The torque required to stop it after 2π revolution is-

$I = 2\text{kg}$ $\qquad \omega = 2\pi f$

$R = 4\text{cm}$ $\qquad \frac{2\pi 3}{60} = \frac{\pi}{10}$ rev/s

$f = \frac{3}{60}$ rus

$\omega_f = 0$

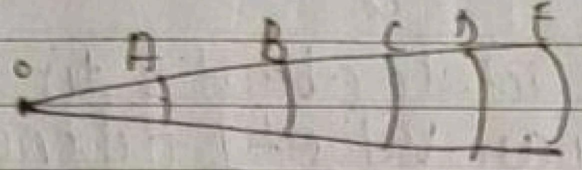
$\theta = 2\pi(2\pi) = 4\pi^2$ rad.

Date
Page

$\alpha = \frac{\omega^2}{2\theta} = \frac{\pi^2}{100 \times 2 \times 4\pi^2}$

$\tau = \frac{MR^2\alpha}{2} = \frac{2 \times 16}{104 \times 2} \times \frac{1 \times}{(\cancel{000}) \times} = 2 \times 10^{-5}$

Rod is in Pure rotation.



θ - same
ω - same
α - same

$V = r\omega$
differ ↘ same

$V_A < V_B < V_C < V_D < V_E$

Q) A rod PQ of mass M and length L is Hinged at end P. The rod is kept Horizontal by a massless string tied to point. when string is cut the initial angular accⁿ of rod is.

$\tau_P = I_P \alpha$

$N \times 0 + Mg\frac{L}{2} = \frac{ML^2}{3}\alpha$

$\alpha = \frac{3g}{2L}$

Q) Find accⁿ of C.O.M and Point Q when Q. when rod is just released, also find Normal rxxⁿ at that point.

$\vec{a} = \vec{a}_T = r\alpha$ $\qquad\qquad \tau = I\alpha$

$a_{cm} = \frac{L}{2}\left(\frac{3g}{2L}\right) = \frac{3g}{4}$ $\qquad mg\frac{L}{2} = \frac{mL^2}{3}\alpha$

$\alpha = \frac{3g}{2L}$

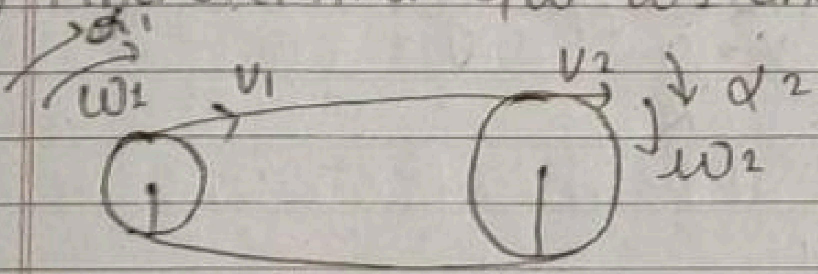
$a_P = 0 \qquad \vec{a}_Q = L\left(\frac{3g}{2L}\right) = \frac{3g}{2}$

$F_{net} = m a_{cm}$

$mg - N = m a_{cm}$

$N = mg - m a_{cm} = mg - m\frac{3g}{4} = \frac{mg}{4}$

Q) find relation b/w ω_1 and ω_2



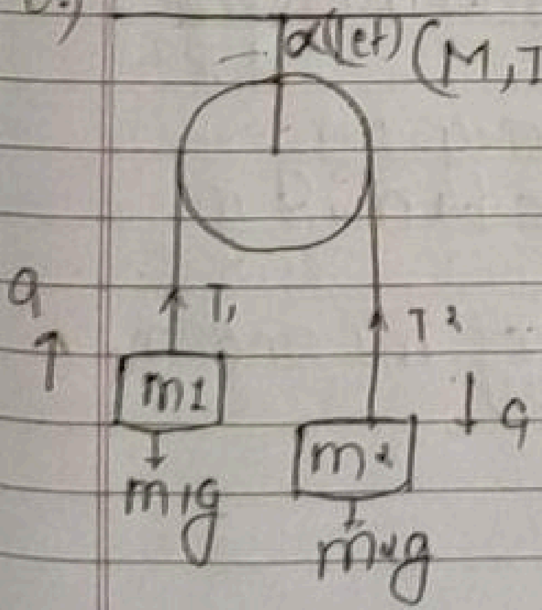
Tangential acc^n equal. $\qquad$ tangential speed same

$a_1 = a_2 \qquad\qquad v_1 = v_2$

$r\alpha_1 = R\alpha_2 \qquad\qquad r\omega_1\uparrow = R\omega_2\downarrow$

$\alpha_1 > \alpha_2 \qquad\qquad \omega_1 > \omega_2$

Q.)



Ist part

FBD of m_2

$m_2 g - T_2 = m_2 a$ (I)

$T_1 - m_1 g = m_1 a$ (II)

$T_2 - T_1 = \frac{Ia}{R^2}$ (III)

Page

$(m_2 - m_1)g = \left(m_1 + m_2 + \frac{I}{R^2}\right)a$

$\vec{a} = \left[\frac{(m_2 - m_1)g}{m_1 + m_2 + \frac{I}{R^2}}\right]$

$T_2 > T_1$ clockwise rotation

for pulley $\tau_0 = I\alpha$

$T_2 R - T_1 R = I\alpha$

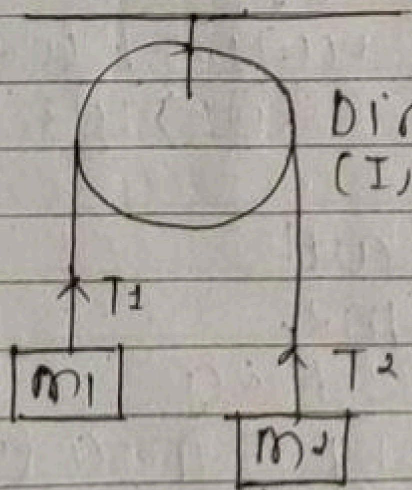
Condition of No slipping

$a = \bar{a}_T$ $\quad \alpha = \frac{a}{R}$

$a = R\alpha$

MR^* $\vec{a} = \frac{F_{net}}{M_{trans} + \frac{I}{R^2}}$

Q)



Disc (I, R, m)

$a = \frac{m_2 g - m_1 g}{m_1 + m_2 + \frac{mR^2}{2R^2}}$

$a = \frac{m_2 g - m_1 g}{m_1 + m_2 + \frac{m}{2}}$

For tension make FBD of m_1 and m_2

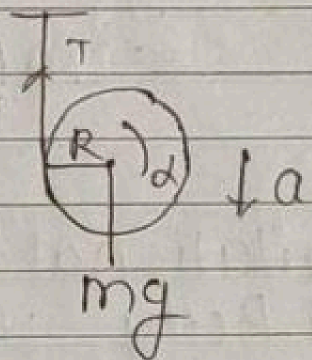
Q) A solid cylinder of M has a string wrapped several times around its circumference. The free end of string is attached to the ceiling and the cylinder is released from rest. Find the accn of the cylinder and the tension in the string.

$$a = \frac{mg}{m + \frac{I}{R^2}} = \frac{mg}{m + \frac{mR^2}{2R^2}} = \frac{2mg}{3m} = 2g/3$$

$$mg - T = ma$$

$$T = mg - \frac{m2g}{3}$$

$$T = \frac{mg}{3}$$



Q) A string is wrapped around the rim of a wheel of moment of Inertia 0.20 kg-m^2 and radius 20cm. The wheel is free to rotate about its axis. Initially the wheel is at rest. The string is now pulled by a force of F = 20N. Find the angular velocity of wheel after 5 sec.

$$\tau = FR = I\alpha$$

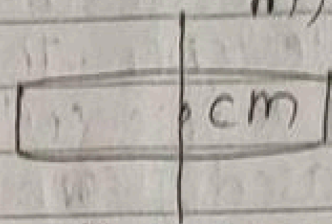
$$\alpha = \frac{FR}{I} = \frac{20 \times 20/100}{\frac{20}{100}} = 20 \text{ rad}/$$

$$w_f = w_i + \alpha t$$

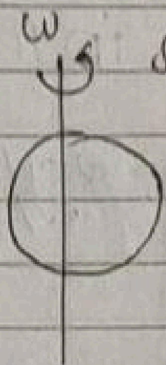
$$= 20 \times 5 = 100 \text{ rad/s}$$

Kinetic energy in pure rotational motion

m, L



$K.E = \frac{1}{2} I\omega^2$



Solid sphere (M, R)

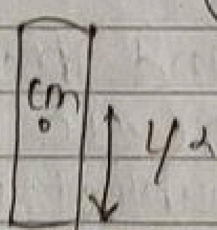
$K.E = \frac{1}{2} I\omega^2 = \frac{mL^2\omega^2}{24}$

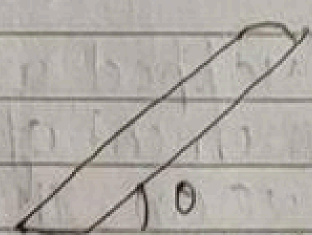
$K.E = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2$

$= \frac{mR^2\omega^2}{5}$

Gravitational Potential energy of rigid body

u(r.o.f. = 0)

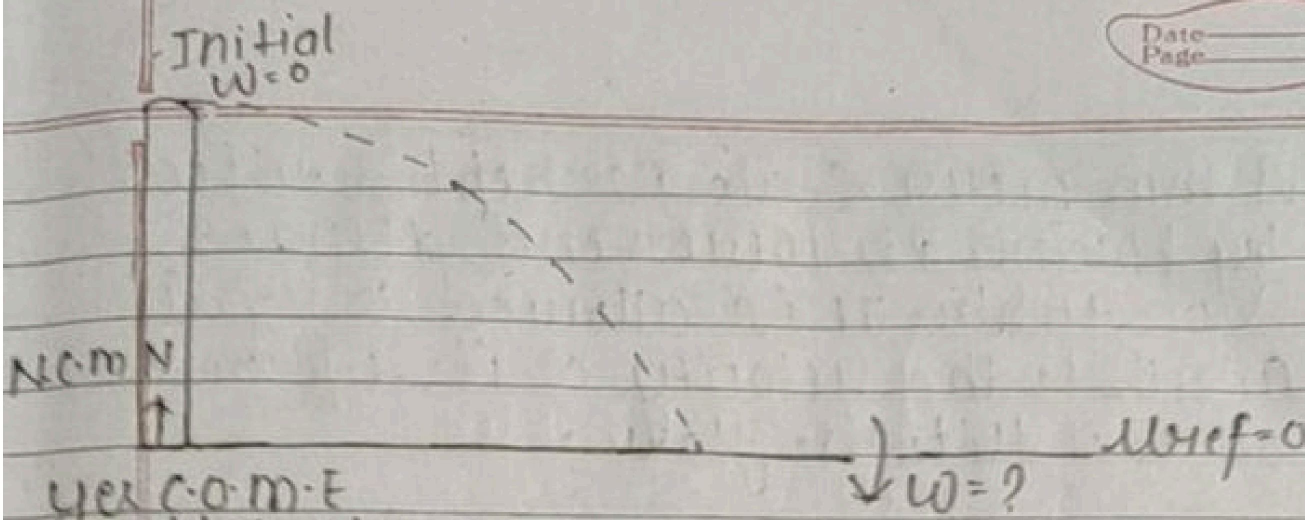




$U = \frac{mgL}{2}$ $U = \frac{mgL}{2}\sin\theta$

$U = -\frac{mgL}{2}$

Conservation of mechanical energy in pure rotational motion



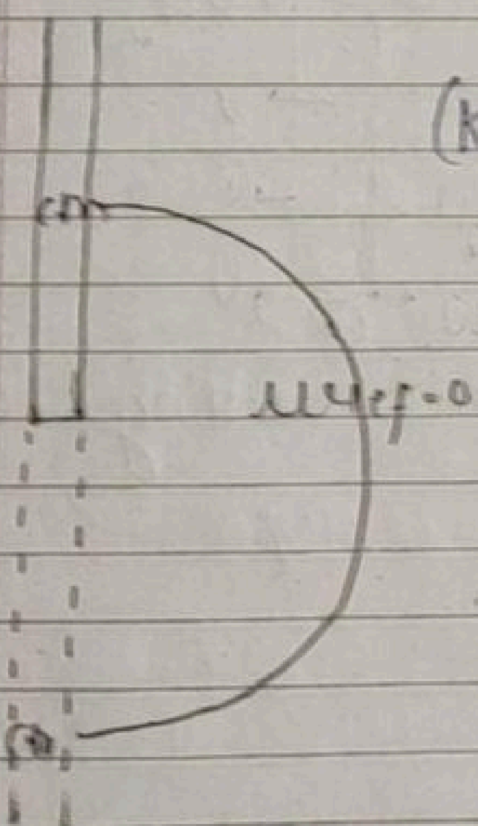
yes C.O.M.E applicable becoz $W_N = 0$

$$[K.E + U]_i = [K.E + U]_{final}$$

$$0 + \frac{mgl}{2} = \frac{1}{2} I\omega^2 + 0$$

$$\frac{\cancel{m}gl}{2} = \frac{1}{\cancel{2}} \frac{\cancel{m}l^2}{3} \omega^2 \qquad \omega = \sqrt{\frac{3g}{l}}$$

Q)



C.O.M.E-

$$(K.E + U)_i = (K.E + U)_f$$

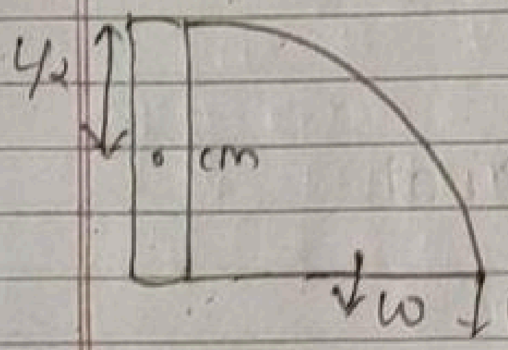
$$0 + \frac{mgl}{2} = \frac{1}{2} I\omega^2 - \frac{mgl}{2}$$

$$2mgl = \frac{ml^2\omega^2}{3}$$

$$\omega^2 = \frac{6g}{l} = \omega^2 = \frac{6g}{l}$$

$\omega = \sqrt{\frac{6g}{l}}$ for all point

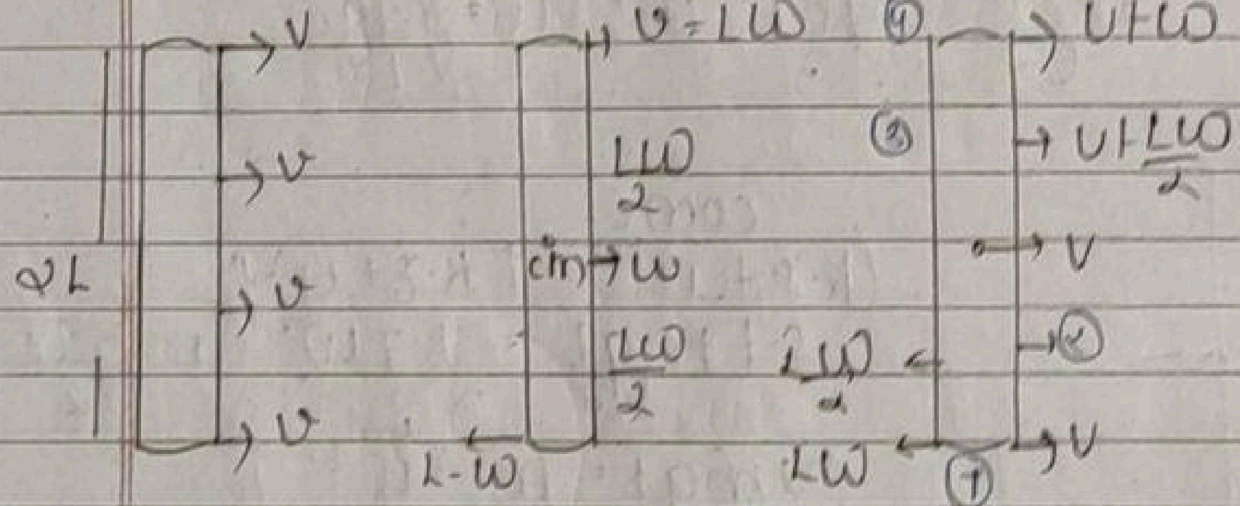
Q) A thin meter scale is kept vertical by placing its lower end hinged on floor. It is allowed to fall. Calculate the velocity of its upper end when it hits the floor.



$\omega = \sqrt{\frac{3g}{L}}$

$V_p = r\omega = l\sqrt{\frac{3g}{L}} = 1\sqrt{\frac{3g}{1}}$

$= \sqrt{3g} = \sqrt{30} = 5.5$ m/s

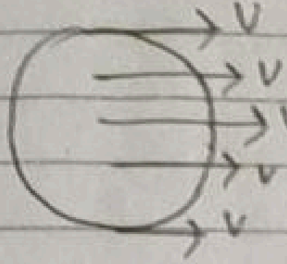


pure Transl^n pure rotation rolling

$v_4 > v_3 > v_{cm} > v_2 > v_1$

Date
Page

(I)

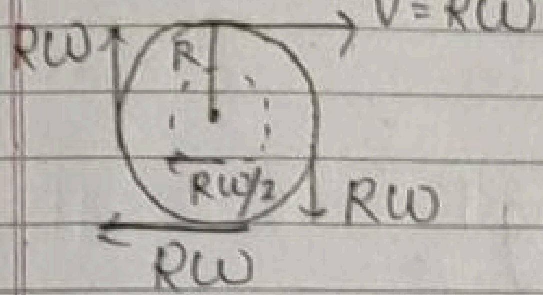


Pure translstⁿ motion

V = same

$K.E = \frac{1}{2} mv^2$

(II)



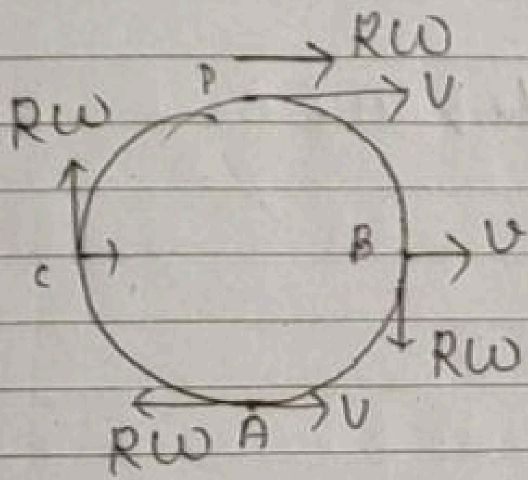
Pure rotational motion

ω = same V = not same

$V_{cm} = 0$

$K.E = \frac{1}{2} I\omega^2$

(III)

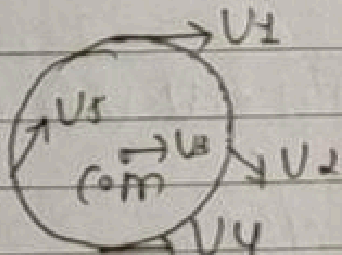


Rolling motion

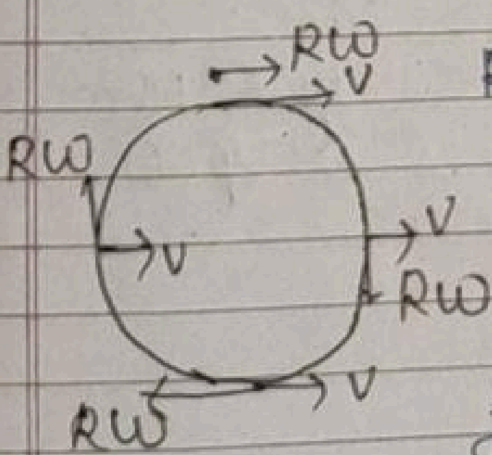
Velocity of c.o.m will be v

$V_D > V_C > V_B > V_{cm} > V_A$

(IV)



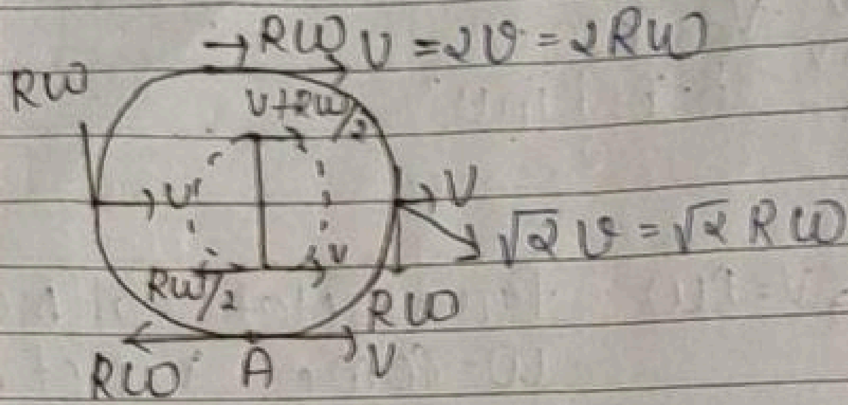
$V_1 \neq V_2 \neq V_3 \neq V_4 \neq V_5$



Rolling motion = Tranlⁿ + Rotⁿ

If $V > R\omega$
tranlⁿ dominating
forward slipping
friction backward

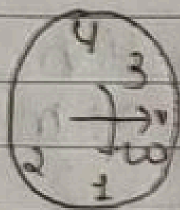
If $V < R\omega$
Rotⁿ do
Back. slip
friction
↓
forward



If $v = R\omega$

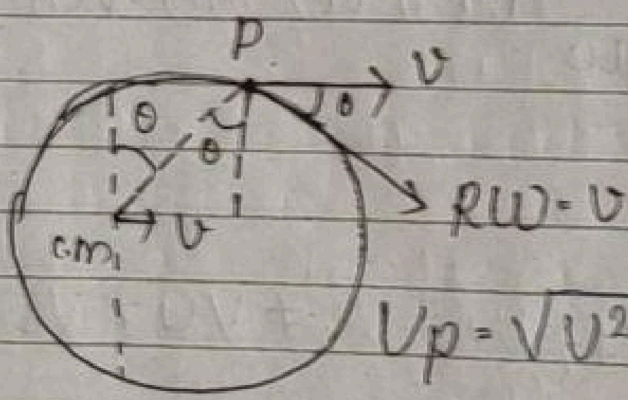
$V_A = 0$ [pure rolling]

No slipping



$V_4 > V_3 > V_2 > V_1$

(I)



$V_p = \sqrt{v^2 + v^2 + 2v \cdot v \cos\theta}$

pure rolling motn. $v_p = 2v\cos(\theta/2)$

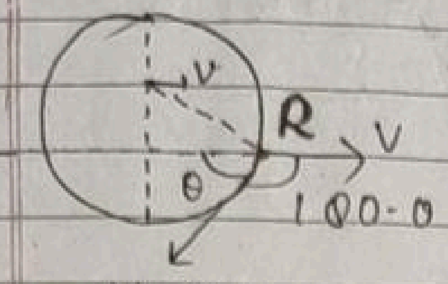
$v = R\omega$

If $\theta = 0°$
$v_p = 2v$

If $\theta = 90°$
$v_p = \sqrt{2}\, v$

If $\theta = 180°$
$v_p = 0$

(II)



$V = R\omega$

$V_R = 2V\cos\left(\frac{180-\theta}{2}\right)$

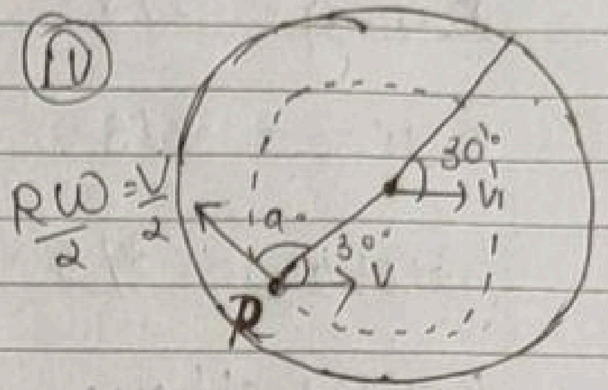
$2V\cos\left(90-\frac{\theta}{2}\right)$

$V_R = 2V\sin(\theta/2)$

Q) Find square root index

$\frac{\sqrt{16}}{2}$ $\sqrt{16} = \sqrt{4} = \sqrt{2}$

(IV)



Pure rolling ($V = R\omega$)

$V_P = \sqrt{V^2 + \left(\frac{V}{2}\right)^2 + 2V\frac{V}{2}\cos(120^\circ)}$

$\sqrt{V^2 + \frac{V^2}{4} + V^2\left(\frac{-1}{2}\right)}$

$V^2 + \frac{V^2}{4} - \frac{V^2}{2} = \sqrt{\frac{4V^2 + V^2 - 2V^2}{4}}$

$V_P = \frac{\sqrt{3}}{2}V$

Rolling -

Total Energy in Rolling motion. $R\omega = V$

$K.E_{total} = K.E_{transl^n} + K.E_{rot^n}$

$K.E_{total} = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}mk^2\left(\frac{V^2}{R^2}\right)$

$\frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right]$

$$100 \times \frac{K.E\ trans^{ln}}{K.E\ total} = \frac{1}{1+\frac{K^2}{R^2}} = \beta \times 100$$

$\beta_{ring} = 1/2 = 0.5$

$\beta_{disc} = 2/3 = 0.66$

$\beta_{Hollow} = 3/5 = 0.6$

$\beta_{solid\ sphere} = 5/7 = 0.71$

$\beta_{ring} < \beta_{Hollow} < \beta_{Disc} < \beta_{solid}$

0.5   0.6   0.66   0.71

Q) when a Body is under pure rolling the fraction of its total K.E which is purely rotational is $\frac{2}{5}$. Identify the body.

$1 - \beta = \frac{2}{5} = 0.4$ Hollow

$\frac{K.E\ transl^n}{K.E\ total} = \beta$ $\frac{K.E\ rota}{K.E\ total} = 1 - \beta$

Date
Page

The kinetic energy of a rolling solid cylinder of mass 4 kg about its axis of rotation is 20 kg m² s⁻². calculate velocity with which it is moving.

Solid cylinder ⇒ Disc $\beta = \frac{2}{3} = 0.66$

$\frac{k.E\ transl^n}{k.E\ total} = \frac{2}{3}$ $\qquad \frac{\frac{1}{\cancel{2}} \cancel{4} v^2}{20} = \frac{\cancel{2}}{3}$ $\qquad v^2 = \frac{20}{3}$

$v = \sqrt{\frac{20}{3}}$

Q) which of the following have max. % of total k.E in rotational form while pure rolling

$\frac{k.E\ tran}{k.E\ \cancel{rotational}\ total} = \beta = 50 \quad 60 \quad 66 \quad 71$

% $\frac{k.E\ rotation}{k.E\ total} = 1 - \beta = 50 \quad 40 \quad 33 \quad 20$

ring

Date
Page

Physical Quantity	Ring Hollow cylinder	Hollow sphere	Disc/ Solid cylinder	Solid sphere
β	0.5	0.6	0.66	0.7
$\frac{K.E_{trans}}{K.E_{total}}$	50%	60%	66%	71
$\frac{K.E_{rotatio}}{K.E_{total}}$	0.5	0.4	0.33	0.2
$\frac{K.E_{tran^n}}{K.E_{rotat^n}}$ $\frac{\beta}{1-\beta}$	1:1	3:2	2:1	5:2
Accn on Inclined $\beta g\sin\theta$	$\frac{g\sin\theta}{2}$	$\frac{3}{5}g\sin\theta$	$\frac{2}{3}g\sin\theta$	$\frac{5}{7}g\sin$

Q) A disc of radius 2m and mass 100 kg rolls on a Horizontal floor. its centre of mass has speed of 20 cm/s. How much work is needed to stop it?

$W = K.E_{total} = \frac{K.E_{trans^n}}{\beta}$ $\frac{3}{2} \times \frac{1}{2} \times 100 \left(\frac{2}{10}\right)^2$

$= \frac{3 \times \cancel{4} \times \cancel{100}}{\cancel{4} \times \cancel{100}} = 3J$

Date
Page

Q) A Solid sphere is in rolling motion. In rolling motion a body possess transl^n K.E (Kt) and rot^n (Kr). the ratio Kt : (Kt + Kr) for sphere.

$$\frac{Kt}{Kt+Kr} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2+\frac{1}{2}I\omega^2} = \frac{mv^2}{mv^2+mk^2\frac{v^2}{R^2}}$$

$$\frac{1}{1+\frac{k^2}{R^2}} = \frac{1}{1+\frac{2}{5}\frac{R^2}{R^2}} = \frac{1}{1+\frac{2}{5}} = \left[\frac{5}{7}\right]$$

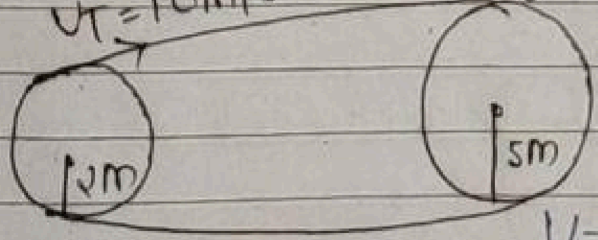
Q) A circular disc of mass 2kg and radius 1 cm rolls without slipping with a speed 2 m/s. the total K.E of disc is.

$v_{cm} = 2m/s$

$\frac{1}{2}mv^2 = \frac{1}{2}2(2)^2 = 4J$

$K.E(total) = \frac{3}{2} \times \cancel{4}^{2} = 6J$

Q) Find Angular speed of both the disc? if speed of connecting string is 10m/s.



$V = r\omega$ | $V = r\omega$

$10 = 2\omega$ | $10 = 5\omega$

$\omega = 5 rad/s$ | $\omega = 2 rad/s$

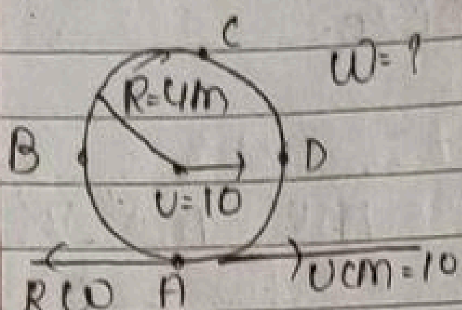
Q) Find ω and velocity of point A, B, C, D if body is in pure rolling motion.

Pure rolling motion

$R\omega = V_{cm}$

$4\omega = 10$

$\omega = 2.5$ rad/sec.



$V_A = 0$

$V_B = 2V_{c.m}$

$V_C = V_D = \sqrt{2}V$

If $V_{c.m} > R\omega$
fr Backward

If $V_{cm} < R\omega$
fr forward

If $V_{cm} = R\omega$
friction = 0

Rolling on moving Surface

Date
Page

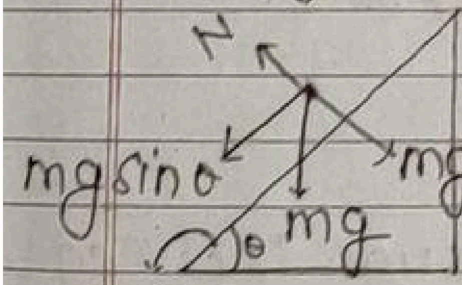
Q) A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the K.E associated with the rotation of the cylinder and total energy.

$$\frac{K.E_{Rot^n}}{K.E_{total}} = 1-\beta \qquad \frac{\frac{1}{2}I\omega^2}{K.E_{total}} = 1-\frac{2}{3}$$

$$\frac{3}{2}I\omega^2 = K.E_{total}$$

$$\frac{3}{2}\frac{mR^2}{2}(100)^2 = K.E_{total}$$

Rolling motion on Smooth Inclined Plane.



$\mu = 0$

$N = mg\cos\theta$

For transl^n motion along the Inclined Plane

$\cancel{m}g\sin\theta = \cancel{m}a_{c.m}$

$a_{c.m} = g\sin\theta$

for all object doesn't depend on shape and size.

Rotation will not start.

for rotational motion about O.

$\tau_0 = mg \times 0 + N \times 0 = 0$

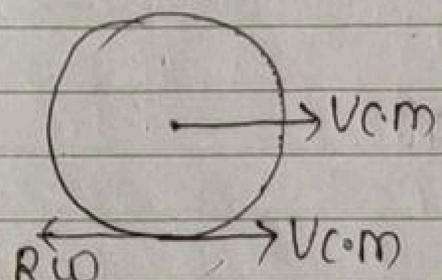
$\alpha = 0$

No rotation No rolling, only transl^n

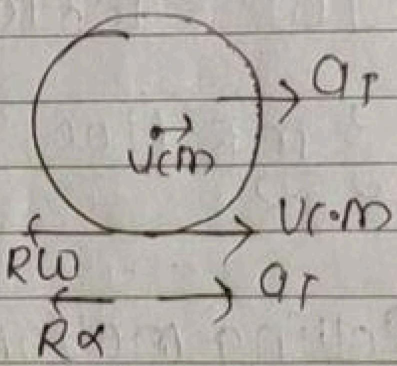
Date
Page

Q) On smooth Inclined, pure rolling motion is possible or not?
Rolling motion on smooth Inclined is possible but to start rolling isn't rolling

On smooth Inclined plane, pure rolling motion can't start.

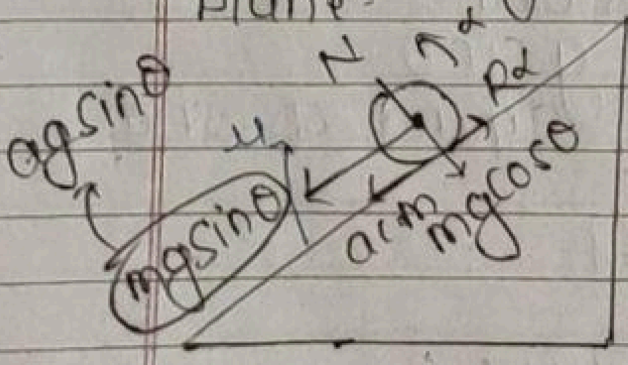


$V_{c.m} = R\omega$
pure rolling
No slipping



$a_T = R\alpha$

Pure rolling motion on rough Inclined plane.



$$a_{c.m} = \frac{mg\sin\theta}{m + I/R^2}$$

Static friction, ω by friction = 0

Date
Page

No slipping

$a_{cm} = R\alpha$ $\quad \alpha = \frac{a_{cm}}{R}$ $\quad mg\sin\theta - f_r = ma_{cm}$

$$f_r = \frac{I a_{cm}}{R^2}$$

$$mg\sin\theta = a_{cm}\left(m + \frac{I}{R^2}\right)$$

for Tran^n motion along inclined

$mg\sin\theta - f_r = ma_{cm}$

Rot^n motion about centre.

$\tau_{cm} = mg\times 0 + N\times 0 + f_r\times R$

$I\alpha = f_r R$

$I\alpha = f_r R \quad f_r = \frac{I\alpha}{R} = \frac{I a_{cm}}{R^2}$

Here no contribution of friction in acc^n, but to start rolling motion there is a role of friction.

workdone by friction (pure rolling) = 0

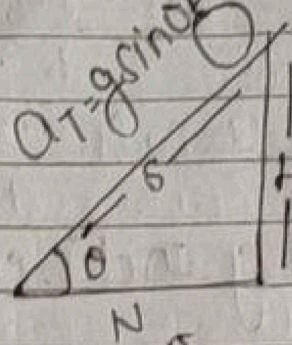
$a_{cm} = \frac{mg\sin\theta}{m + \frac{I}{R^2}}$ doesn't depend on friction

$a_{cm} = \beta(g\sin\theta)$

$t_{ring} > t_{Hollow} > t_{disc} > t_{Solid}$

Date
Page

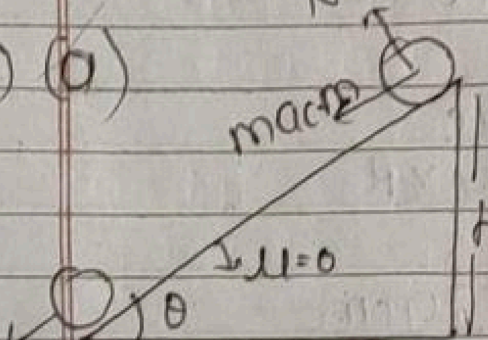
Q) Find time taken to ground.



$s = \cancel{ut} + \frac{1}{2}at^2 = \frac{1}{2}g\sin\theta\beta t^2$

$t = \sqrt{\frac{2s}{g\sin\theta\beta}} = \sqrt{\frac{2H}{g\sin^2\theta\beta}}$

(I) (a)



Vcm=?

C.O.M.E.

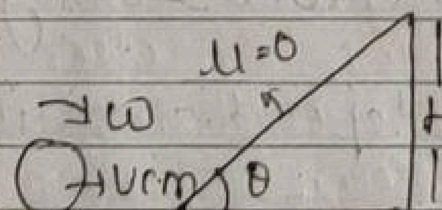
$(K.E + U)_{initial} = (K.E + U)_{final}$

$mgh + 0 = 0 + \frac{1}{2}mv^2$

$V_{cm} = \sqrt{2gh}$

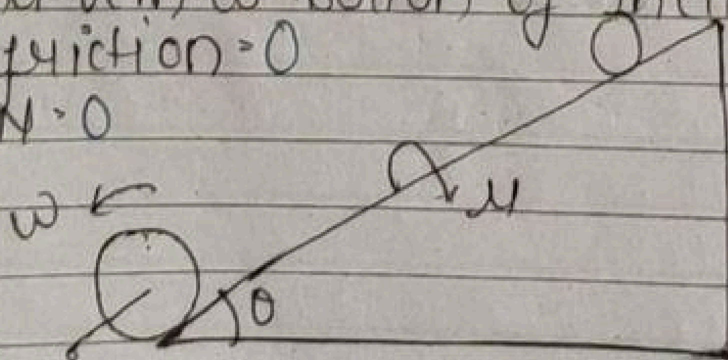
$\beta = \frac{1}{1 + \frac{K^2}{R^2}}$

(b)



$H_{max} = \frac{V_{cm}^2}{2g}$

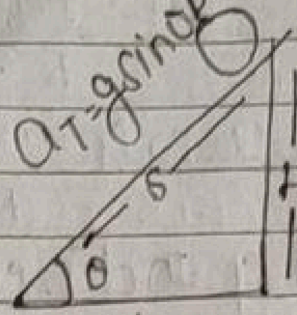
(II) (a) Body rolls without sliping then find vcm at bottom of Inclined.
Find vcm at bottom of Inclined.
$W_{friction} = 0$
$W_N = 0$



$U_{ref} = 0$

Date
Page

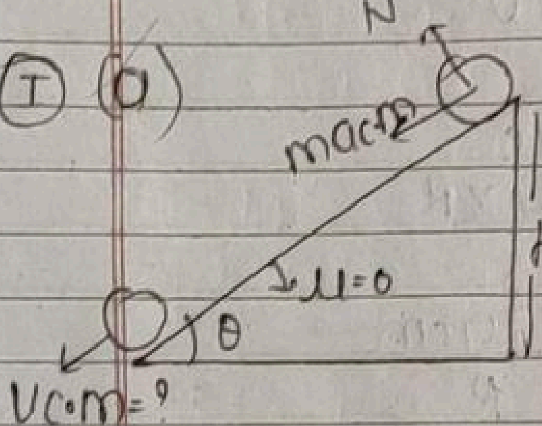
Q) Find time taken to ground.



$S = \cancel{ut} + \frac{1}{2}at^2 = \frac{1}{2}g\sin\theta\beta t^2$

$t = \sqrt{\frac{2S}{g\sin\theta\beta}} = \sqrt{\frac{2H}{g\sin^2\theta\beta}}$

(I) (a)

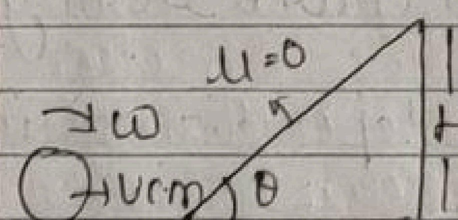


C.O.M.E.
$(K.E + U)$ initial $= (K.E + U)$ final
$mgh + 0 = 0 + \frac{1}{2}mv^2$

$V_{cm} = \sqrt{2gh}$

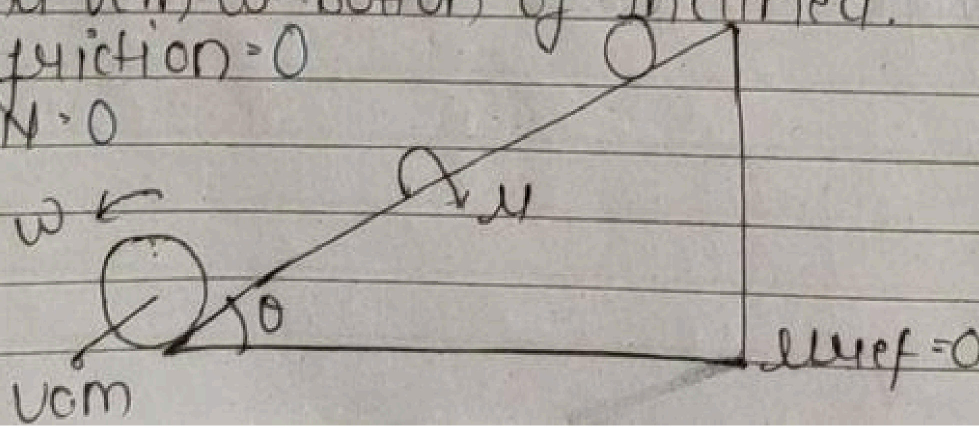
$\beta = \frac{1}{1 + \frac{K^2}{R^2}}$

(b)



$H_{max} = \frac{V_{cm}^2}{2g}$

(II) (a) Body rolls without sliping then find V_{cm} at bottom of Inclined.
Find V_{cm} at bottom of Inclined.
$W_{friction} = 0$
$W_N = 0$

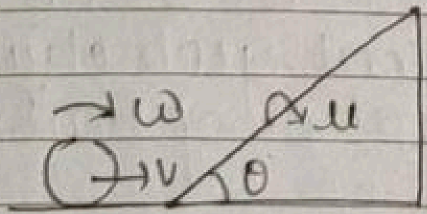


C.O.M.E

$mgh + 0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + 0$

$mgh = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)$ $\quad v_{c.m} = \sqrt{\beta 2gH}$

(b)



$H_{max.} = \frac{v^2}{2g\beta}$

Q) Two solid sphere of different mass, radii and density roll down a rough Inclined plane under Identical situation. Their time to come down is Independent of -
mass, Radius, Density

A small object of uniform density rolls up a curved with an Initial Velocity v. It reaches upto a max Height of $\frac{3v^2}{4g}$ with respect to the Initial position. The object is

$H_{max} = \frac{v^2}{2g\beta} = \frac{3v^2}{4g}$ $\quad \beta = \frac{2}{3} = 0.66$ Disc

Q) A solid sphere and a solid cylinder having same mass and radius, roll down the same Incline. The ratio of their accn will be

$$\frac{a \text{ solid sphere}}{a \text{ solid cylinder}} = \frac{\beta_{solid}}{\beta_{s.cyli}} = \frac{5 \times 3}{7 \times 2} = \frac{15}{4}$$

Q) A thin circular ring first slips down a smooth Incline then rolls down a rough Incline of Identical geometry from same Height. ratio of time ta in the two motion.

$a_1 t_1^2 = a_2 t_2^2$

$g \sin\theta \, t_1^2 = \frac{g \sin\theta}{2} t_2^2$

$\frac{t_1}{t_2} = \frac{1}{\sqrt{2}}$

⇒ when a body is rolling without slipi on a rough Horizontal surface, the work done by friction is always zero.

Q) Friction force action on rough Inclined plane on pure rolling mo

for Tranlⁿ motion.

$mg\sin\theta - f_r = Ma_{c.m}$

$f_r \times R = I\alpha$

$f_r = \frac{I\alpha}{R} = \frac{I a_{c.m}}{R^2}$

put value of a.c.m $= \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$ in fr.

$$fr = \frac{mg\sin\theta}{\left(\frac{R^2}{K^2}+1\right)} = mg\sin\theta(1-\beta)$$

Minimum co-efficient of friction to start pure rolling motion.

$(fr)_{limiting} = \mu N = \mu mg\cos\theta$

$(fr)_{static} > f_{limiting}$ slipping will occur

$(fr)_{static} \le (fr)_{lim}$

$mg\sin\theta(1-\beta) \le \mu mg\cos\theta$

$\tan\theta(1-\beta) \le \mu$

$\mu_{min} = \tan\theta(1-\beta)$

Q) what is the min. coefficient of friction for a solid sphere to roll without slipping on an inclined plane of inclination θ?

$\mu_{min} = \tan\theta(1-\beta)$

$\frac{2}{7}\tan\theta$

Q) A cylinder of radius R and M rolls without slipping down a plane inclined at an angle θ. Coefficient of friction b/w the cylinder and the plane is μ for what max. inclination θ will by cylinder roll without slipping?

$\mu = (1-\beta)\tan\theta = \left(1 - \frac{2}{3}\right)\tan\theta$

$3\mu = \tan\theta$

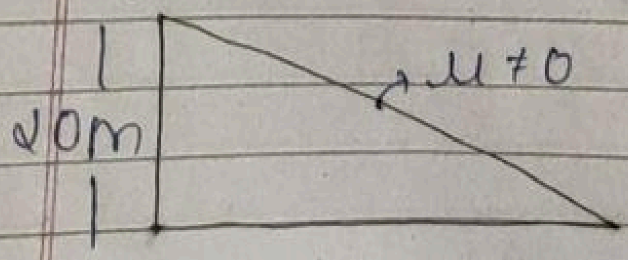
$\theta = \tan^{-1}(3\mu)$

Q) Two disc having masses in the ratio 1:2 and radius 1:8 roll down without slipping one by one from an inclined plane of height h. The ratio of their linear velocities on reaching the ground is.

$m_1 : m_2 = 1:2$ $\quad \frac{v_1}{v_2} = \frac{\sqrt{2\cancel{gh}\beta}}{\sqrt{2\cancel{gh}\beta}} = \frac{1}{1}$

$R_1 : R_2 = 1:8$

Q) A solid sphere is rolling down an inclined plane without slipping of height 20m. Calculate the max. velocity with which it will reach the bottom of plane



$v = \sqrt{2gh}$

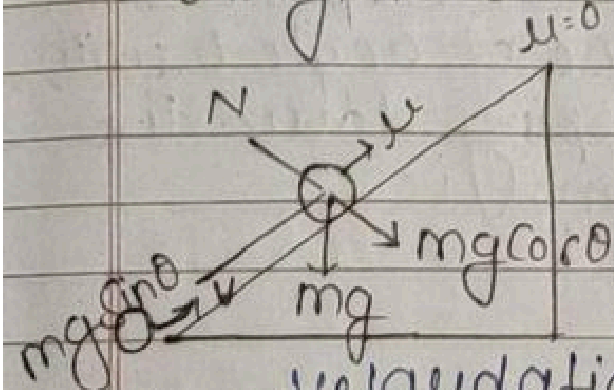
$= \sqrt{\frac{2 \times 10 \times 20 \times 5}{7}} = \sqrt{\frac{20\ldots}{7}}$

Date
Page

Q) The speed of a uniform solid cylinder of rolling down an Inclined plane of Vertic Height H, from rest without sliding is

$v = \sqrt{2gh \cdot B} = \sqrt{2gH \times \frac{2}{3}} = \sqrt{4gH}$

Q) Find max Height H object. Object is in pu rolling motion.



C.O.M.E.

$0 + \frac{1}{2}mv^2 + \frac{I}{2}\omega^2 = mgH +$

$H = \frac{v^2}{2g}$

retardation produced by $mg\sin\theta$.
Speed $v \downarrow$

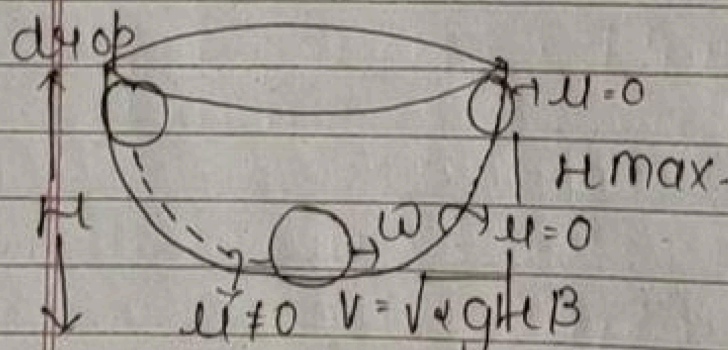
$\tau_{c.m} = 0$
$\alpha = 0$
$\omega = \text{const}^n$

Q) A Solid Sphere is thrown up a rough Incline. The sphere rolls up without slip and eventually comes down rolling with slipping. The direction of rolling friction in upward and downward motion

upward, upward

Q) The ratio of acc^n for a solid sphere rolling down an Incline of angle θ without slipping and slipping down the Incline without rolling is.

$$\frac{a_1}{a_2} = \frac{\cancel{g\sin\theta}\,\beta}{\cancel{g\sin\theta}} = \frac{5}{7}$$



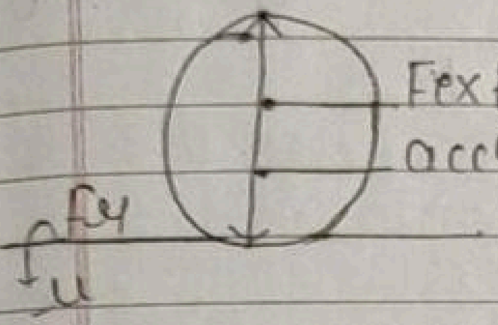
$$H_{max} = \frac{V^2}{2g} = \frac{2gH\beta}{2g} = H\beta = \frac{3H}{5}$$

Rolling motion on Horizontal surface. Find accn of c.o.m and angular accn if body is in pure rolling motion.

For transln.

$F - F_r = m a_{c.m}$

$\tau_0 = I\alpha \qquad a_{c.m} = R\alpha$



$Fh + f_r R = \frac{I a_{cm}}{R}$

$\frac{Fh}{R} + f_r = \frac{I a_{cm}}{R^2}$

$\boxed{F - f_r = m a_{c.m}}$

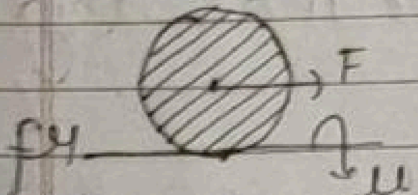
$a_{c.m} = \frac{F\left(1 + \frac{h}{R}\right)}{m + \frac{I}{R^2}}$ μ

If $h = 0$

$a_{c.m} = \frac{F}{\left(m + \frac{I}{R^2}\right)}$

$h = R$

$a_{c.m} = \frac{2F}{m + I/R^2}$



$a = \frac{F}{M} \times \frac{2}{3} = \frac{2F}{3m}$

Disc (M, R) $\quad \alpha = \frac{a}{R} = \frac{2F}{3mR}$

$F - f_r = m\left(\frac{2F}{3m}\right)$

$F - \frac{2F}{3} = f_r$

$\frac{F}{3} = f_r$

Q) An object is rolling without slipping on a Horizontal surface and its rotational kinetic energy is $\frac{2}{3}$ of translational K.E. The body may

$$K.E_{rotation} = \frac{2}{3} K.E_{transl^n}$$

$$\frac{K.E_{rotation}}{K.E_{transl^n}} = \frac{2}{3} = \frac{1-\beta}{\beta}$$

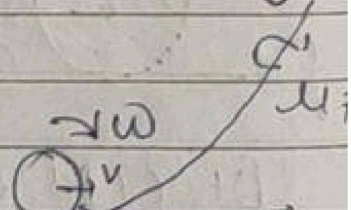
$$2\beta = 3 - 3\beta$$

$$5\beta = 3$$

$$\left[\beta = 3/5\right] \Rightarrow \text{Spherical shell}$$

Q) A disc of mass m and radius r rolls on a Horizontal plane without slippin with speed u. if it starts climbing upon Inclined plone, the max Height it would attain will be.

$$H_{max} = \frac{u^2}{2g\beta} = \frac{u^2}{2g\left(\frac{2}{3}\right)} = \frac{3v^2}{4g}$$



Q) An Initial momentum is Imparted to a Homogenous sphere, as a result of which it begins to roll without slip up an Inclined plane at a speed of = 10m/s. The plone makes an angle θ with Horizontal, what Height H will the sphere rise to ?

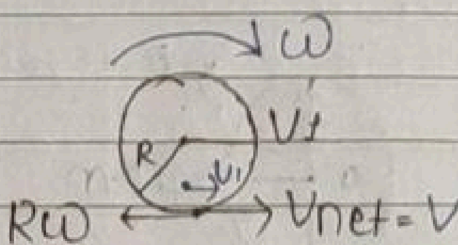
$$H = \frac{v^2}{2g\beta} = \frac{\cancel{100}^{\,5}}{\cancel{2}\times\cancel{10}\times\frac{\cancel{5}}{7}} = 7\,m$$

A body is rolling without slipping on a moving plank. relation b/w v and v_1

for pure rolling

$v_1 - R\omega = v$

$v_1 - v = R\omega$



A solid sphere is rolling on a frictionless surface with a translational velocity v m s^{-1}. if it is to climb the Inclined surface then v should be.

frictionless $h < \frac{v^2}{2g}$ $\quad v = \sqrt{2gh}$

A solid cylinder is roling down on an Inclined plane of angle θ. The Coefficient of static friction b/w the plane and cylinder is μ_s, then condition for the cylinder not to slip is

$f_r(max) = \mu mg\cos\theta$

$(f)\,static = mg\sin\theta\,(1-\beta)$

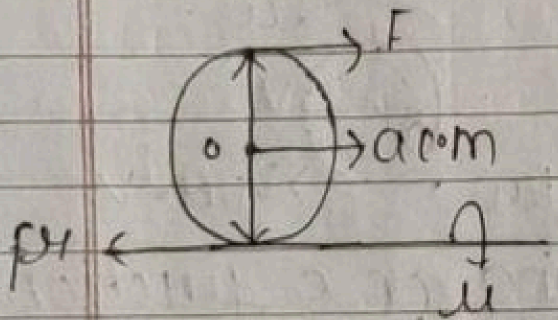
$fr(max) \geq fr(static)$

$\mu mg\cos\theta \geq mg\sin\theta(1-\beta)$

$\mu = \tan\theta(1-\beta)$

Pure rolling on Horizontal Surface-

If this solid sphere is in pure rolling motion due to F force



Transl^n-

$F - fr = ma_{c.m}$

$F + fr = \frac{I a_{c.m}}{R^2}$

$2F = a_{c.m}\left(m + \frac{I}{R^2}\right)$

for rot^n about 'O'

$FR + frR = I\alpha$

$F + fr = \frac{I}{R}\left(\frac{a}{\alpha}\right)$

$a_{c.m} = \frac{2F}{m + I/R^2} = \frac{2F}{m + \frac{2}{5}}$

$a_{c.m} = \frac{10F}{7m}$

$a_{cm} = \frac{2F}{m} \times \beta \qquad F - fr = ma$

Q.) A force F is applied on a Hollow cyl for pure accelerated rolling, the force of friction will be towar

Date
Page

$$a_{c.m} = \frac{2F \times B}{m} = \frac{\cancel{2}F}{m} \times \frac{1}{\cancel{2}} = \frac{F}{m}$$

for Transl^n.

$$F - f_r = m a_{c.m}$$

$$F - m a_{c.m} = f_r$$

$$0 = F - \cancel{m}\left(\frac{F}{\cancel{m}}\right) = f_r \text{ zero}$$

Q) A force F is applied on a Disc. for pure accelerated rolling, the force of friction will the towards.

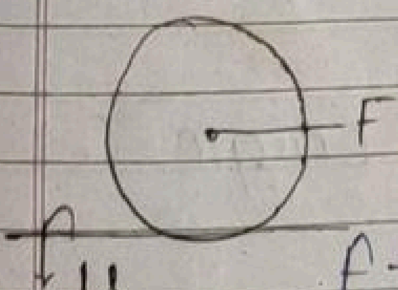
$$a = \frac{2F \times B}{m} = \frac{2F}{m} \times \frac{2}{3} = \frac{4F}{3m}$$

Transl^n.

$$F - f_r = \cancel{m}\left(\frac{4F}{3\cancel{m}}\right) \quad f_r = F - \frac{4F}{3} = -\frac{F}{3}$$

forward

Q) A Hollow sphere place on rough surface then find acc^n and value of friction.



$$a = \frac{F}{m} \times \frac{3}{5} = \frac{3F}{5m}$$

$$F - f_r = ma$$

$$f_r = F - m\left(\frac{3F}{5m}\right) = +\frac{2F}{5} \text{ Backward}$$

only if magnitude of position vector is changing then body is in pure transl motion.

only if direction of position vector is changing then body is in pure rotn.

Angular Momentum -
Quantity of Rotational motion contain in a body is called angular momen

Angular momentum about origin $\perp$ to the plane

$\vec{L} = \vec{r} \times \vec{p}$ $\qquad L \perp P$

$L = rp\sin\theta = mvr\sin\theta$ $\qquad L \perp R$

$\vec{L} \cdot \vec{P} = 0$

In pure circular motion

$L = I\vec{\omega}$

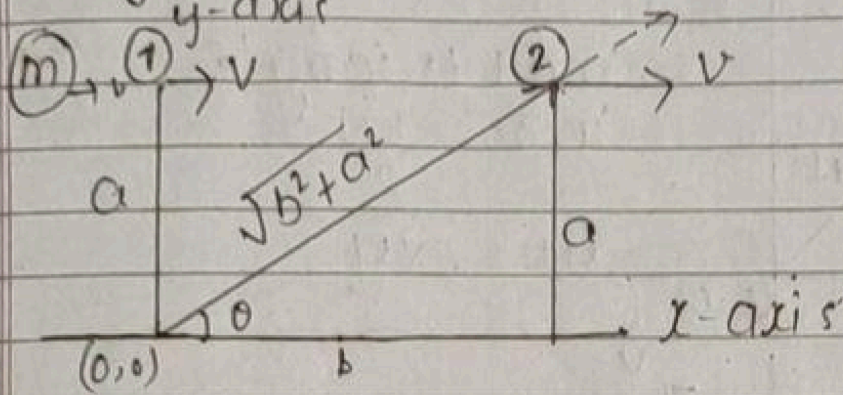
axial vector

dirn along $\vec{\omega}$

unit $kg\,m^2\,rad/s$

Angular momentum about origin = 0

object is moving on straight line, with velocity v then find angular moment about origin.



$(L_1)_{0,0} = r \times P$

$= a m v \sin 90^\circ$

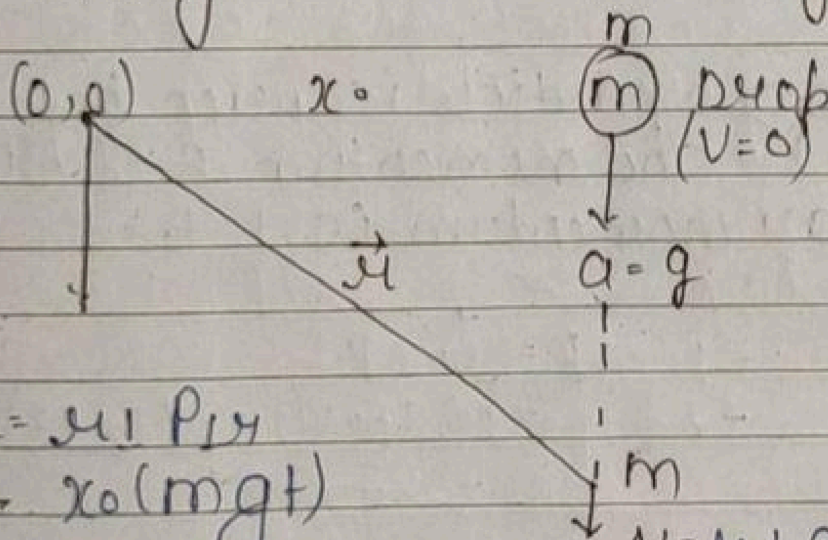
$= mva$

$(L_2)_{0,0} = r P \sin\theta$

$\sqrt{a^2+b^2} \times mv \left(\frac{a}{\sqrt{a^2+b^2}}\right)$

$= mva$

Find angular momentum. After time 't'



$L = r_\perp P_{\perp r}$

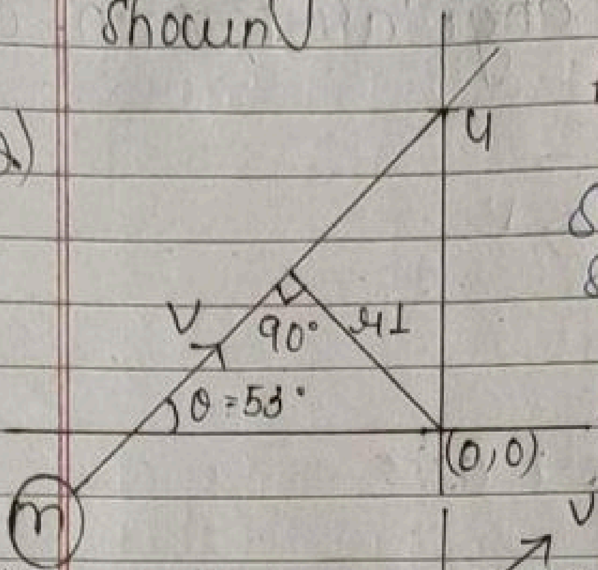
$L = x_0 (mgt)$

$v = u + at = 0 + gt$

$v_y = gt$

Q) Find Angular momentum of object as shown

Q)



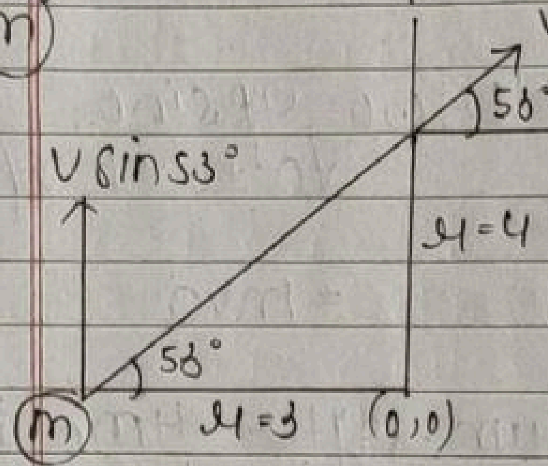
$\tan\theta = \frac{4}{5}$ $\theta = 53°$

Small triangle

$\sin 53° = \frac{r_\perp}{3}$

$r_\perp = 3 \times \frac{4}{5}$

Q)



$L = mv \times \frac{4}{5} \times 3 = \frac{12mv}{5}$

Q) The position of a particle is given by $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k})$ and momentum $\vec{P} = (3\hat{i} + 4\hat{j}$ The angular momentum is $\perp$ to.

$\hat{i}$	$\hat{j}$	$\hat{k}$
1	2	-1
3	4	-2

$L = \vec{r} \times \vec{P}$

$= \hat{i}(-4+4) - \hat{j}(-2+3) + \hat{k}($

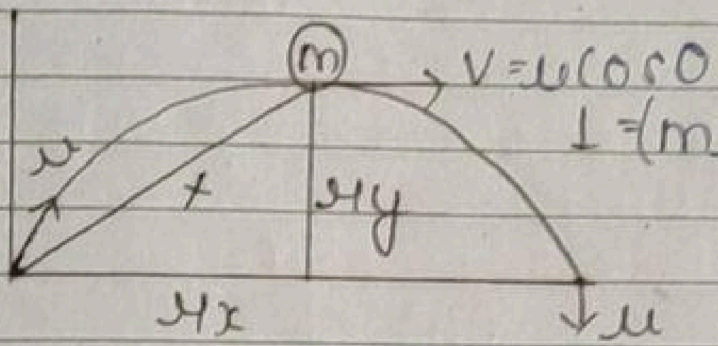
$= -\hat{j} - 2\hat{k}$

Q) A particle of mass m is rotating in a plane in circular path of radius r. its angular momentum is L. The centripe force acting on the particle is

Date
Page

$$F_c = \frac{mv^2}{r} = \frac{mL^2}{rm^2r^2} = \frac{L^2}{mr^3}$$

Q) object is projected with speed u at an angle θ then find angular momentum after time 't' when it is at max. Height.



Q) when a mass is rotating in a plane about a fixed point, its angular momentum is directed along.
The axis of rotation

Q) object is projected with speed u at an angle θ then find angular momentum after time 't' when it is

$L = R\,mu\sin\theta$

$$\frac{u^2\sin(2\theta)\,mu\sin\theta}{g} = 2$$

Date
Page

Q) Ball is projected with u at angle θ th... find Angular momentum at time 't' about point of projection

Torque, $\tau = \frac{dL}{dt}$

unit of angular momentum is Nm-sec

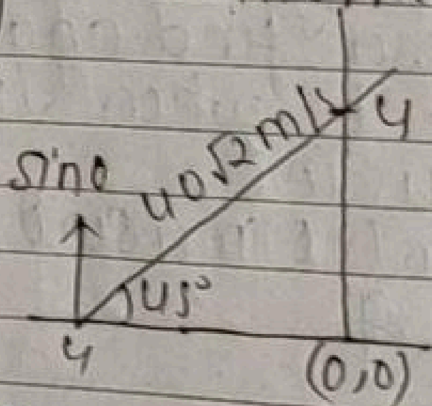
Q) object is moving with velocity $40\sqrt{2}$ m/s on a straight line of equation $y = x+4$ then find angular momentum about origin

$y = mx + c$

$c = 4$

$\tan\theta = m = 1$

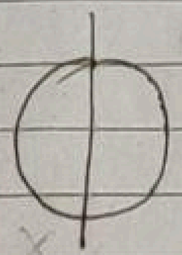
$\theta = 45°$



$40\sqrt{2} \sin 45° = 40m$

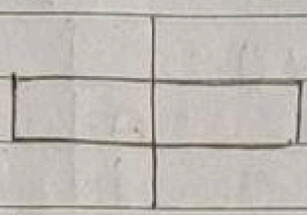
$L = r \times p = 4 \times 40m = 160m$

Angular moment in pure Rotational motion of rigid body.



$L = I\omega$

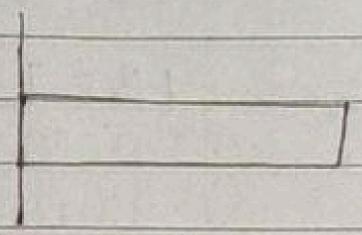
$L = \frac{2}{3} m R^2 \omega$



$L = I\omega$

$L = \frac{m l^2 \omega}{12}$

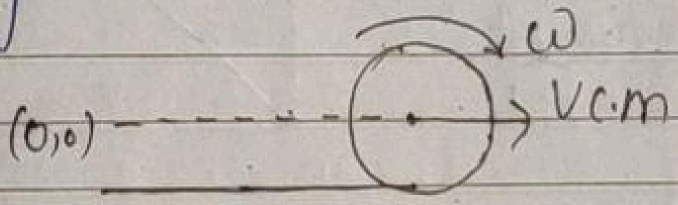
$\frac{m l^2 \omega}{12}$



$L = I\omega$

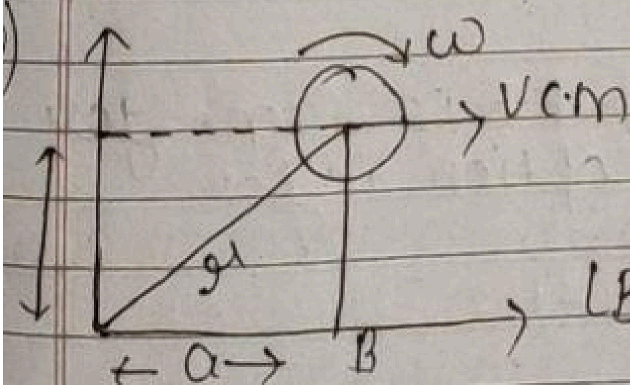
$L = \frac{1}{3} m l^2 \omega$

Angular momentum in transl^n + rot^n = rolling



$L(0,0) = L_{rot^n} + L_{Tran^n}$

$= I_0\omega + 0 = I_0\omega$



$L_A = I_{cm}\omega + 0$

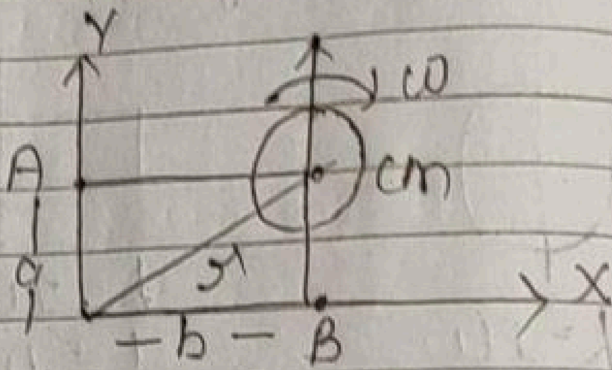
$L_A = I_{cm}\omega \otimes + m V_{cm} b \otimes$

$L_B = I_{cm}\omega \otimes + m V_{cm} b \otimes$

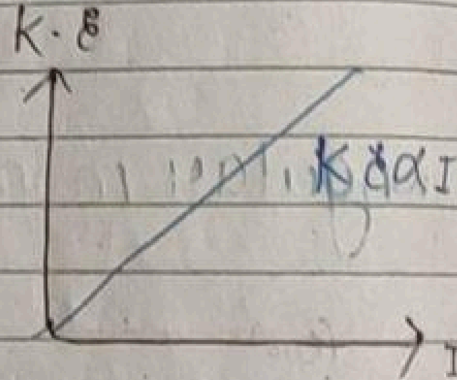
$L_A = I_{cm}\omega \otimes + mvb \odot$

$L_{(0,0)} = I_{cm}\omega \otimes + mvb \odot$

$L_B = I_{cm}\omega + 0$

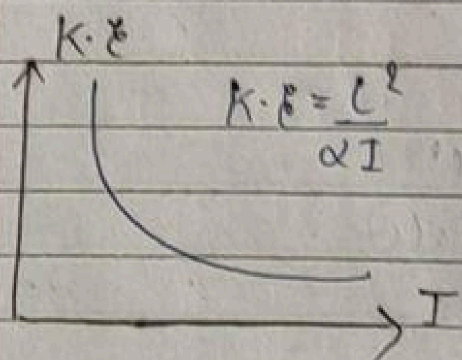


Angular momentum and K.E in pure rolling motion:



$K.E = \frac{1}{2} I\omega^2$ $\quad L = I\omega$

$K.E = \frac{1}{2} I\omega^2 \times \frac{I}{I} = \frac{L^2}{2I}$



If MOI $I_1 > I_2$ and K.E is same for both then correct option is for angular momentum:

$K.E = \frac{L^2}{2I}$ $\quad L^2 \propto I$

same $\quad L_1 > L_2$

Date
Page

Conservation of Linear momentum

If $F_{ext} = 0$ then P will be conserved

Conservation of angular momentum
If $\tau_{ext} = 0$ is zero then $\vec{L}$ will be conserved

ext force may or may not be zero

we have discussed that if the external torque is zero, angular momentum of the system is conserved as $\tau_{net} = 0$

$$\vec{\tau} = \frac{d\vec{L}}{dt} = 0 \quad L = \text{constant}$$

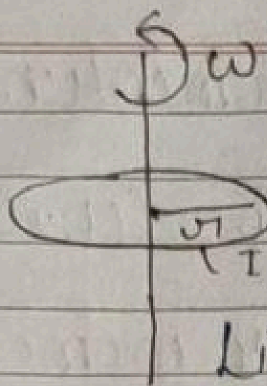
$$I\omega = \text{constant} = L$$

$$I_1\omega_1 = I_2\omega_2$$

Q) A swimmer while jumping into river from a height easily forms a loop in air

He pulls his arm and legs in

Q) A Horizontal disc rotating freely about a vertical axis through its centre makes 90 rev per min. A small piece of wax of mass m falls vertically on the disc and sticks to it at a distance r from the axis. If the no. of revolution per min reduce to 60, then the MOI of disc is



ω $f_1 = 90$ rev per min

$f_2 = 60$ revs per min

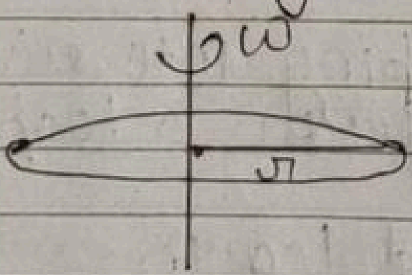
$L_i = L_f$

$I \, 2\pi f_1 = (I + mr^2) \times 2\pi f_2$

$I \, \cancel{90}^{3} = (I + mr^2) \, \cancel{60}^{2}$

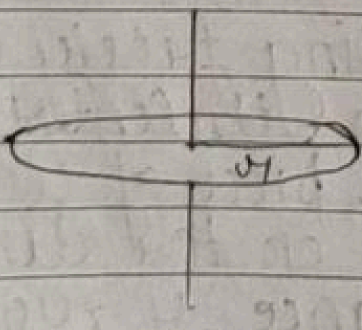
$I = 2mr^2$

a) A thin circular ring of mass m and radius r is rotating about its axis with constant angular velocity ω. Two object each of mass m are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with angular velocity given by.



m

$L_i = I\omega = mr^2\omega$



$L_f = (mr^2 + 2mr^2)\,\omega_f$